# STEMM

# ABSTRACTS

of Uzbek-Israel joint international conference
**S**cience - **T**echnology - **E**ducation - **M**athematics - **M**edicine

**Bukhara - Samarkand - Tashkent**

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National University of Uzbekistan
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# A B S T R A C T S

of the Joint International Conference

# S T E M M

**S**cience – **T**echnology – **E**ducation – **M**athematics – **M**edicine

**May 13–17, 2019**

**BUKHARA – SAMARKAND – TASHKENT**

**STEMM: S**cience – **T**echnology – **E**ducation – **M**athematics – **M**edicine. Abstracts of the Joint International Conference (May 16–17, 2019, Tashkent).

STEMM stands for Science, Technology, Education, Mathematics and Medicine and its purpose is to bring together scientists and experts from various significant areas to present and discuss innovations and future frontiers directions for inspire cooperation and practical advancement in these areas.

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# MATHEMATICS

# Plenary Lectures

## SOLUTION OF THE PROBLEM OF GENERALIZED LOCALIZATION FOR SPHERICAL PARTIAL SUMS

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Historically progress with solving the Luzin conjecture has been made by considering easier problems. For multiple Fourier series one of such easier problems is to investigate convergence almost everywhere of the spherical sums on $T^N \backslash suppf$ (so called the generalized localization principle).

For the spherical partial integrals of multiple Fourier integrals the generalized localization principle in classes $L_p(R^N)$ was investigated by many authors. In particular, in the remarkable paper of A. Carbery and F. Soria the validity of the generalized localization was proved in $L_p(R^N)$ when $2 \leq p < 2N/(N-1)$.

In this paper the generalized localization principle for the spherical partial sums of the multiple Fourier series in $L_2$ - class is proved. It was previously known that the generalized localization was not valid in classes $L_p(T^N)$ when $1 \leq p < 2$. Thus the problem of generalized localization for the spherical partial sums is completely solved in classes $L_p(T^N)$, $p \geq 1$ : if $p \geq 2$ then we have the generalized localization and if $p < 2$, then the generalized localization fails.

## LOCAL DERIVATIONS ON MEASURABLE OPERATORS AND COMMUTATIVITY

**Ayupov Sh. A.**

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Given a Hilbert space $H$ let $B(H)$ be the *-algebra of all bounded linear operators on $H$. Consider a von *Neumann algebra* $M$, i.e. a *-subalgebra of $B(H)$, containing the identity operator, and closed in the weak operator topology. Denote by $S(M)$ the *-algebra of all closed unbounded linear operators on $H$, which are ¥measurable¥ with respect to $M$. A linear operator $d$ on $S(L)$ is called a *derivation* if it satisfies the Leibniz identity $d(xy) = d(x)y + xd(y)$ for all $x, y$ from $S(L)$. A linear operator $D$ on $L(S)$ is called a *local derivation* if for arbitrary $x$ from $S(L)$ there exist a derivation $d_x$ depending on $x$ such that $D(x) = d_x(x)$.

*The first problem* which is discussed in the talk is a description of all derivations on the algebra $S(M)$. We recall that this long standing problem (almost 20 years) is solved for all types of von Neumann algebras except the type $II_1$ case.

*The second problem* related to a problems considered by R.Kadison and is devoted to local derivations on $S(M)$. We show that if $M$ has no direct abelian summands then every local derivation is a (global) derivation. In the case of abelian von Neumann algebra $M$ we give necessary and sufficient conditions on the algebra $M$ to admit local derivations which are not derivations.

Finally, *the third problem* arises from a discussion of local derivations on algebras with professor Efim Zelmanov in California University, San Diego. Earlier we proved that in the abelian case the square $d^2$ of each derivation on $S(M)$ is a local derivation. Naturally, the question arises whether this property characterizes abelian algebras. In this talk we show that this conjecture is true for algebras $S(M)$, but not for arbitrary algebras. Namely, we prove that the square of every derivation on $S(M)$ is a local derivation if and only if the von Neumann algebra $M$ is commutative. We also give an example of non-commutative associative algebra on which the square of every derivation is a local derivation.

## ISOMETRIES OF IDEALS OF COMPACT OPERATORS

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Let $H$ be a complex separable Hilbert space, and let $K(H)$ be the $C^*$-algebra of all compact linear operators on $H$. Let $C_p = x \in K(H) : Tr(|x|_p < \infty$ be $p$-th Schatten ideal of compact operators with the norm $||x||_p = (Tr(|x|_p))^{1/p}, 1 \leq p < \infty$. In 1975, J. Arazy gave the following description of all the surjective isometries of Schatten ideals $C_p$ : If $V : C_p \to C_p, 1 \leq p < \infty, p \neq 2$, is a surjective isometry, then there exist unitary operators $u_1$ and $u_2$ or anti-unitary operators $v_1$ and $v_2$ on $H$ such that either $Vx = u_1 \times u_2$ or $Vx = v_1 \times^* v_2$ for all $x \in C_p$. The Schatten ideals $C_p$ are examples of Banach symmetric ideals $(C_E, ||.||_{CE})$ of compact operators associated with symmetric sequence spaces $(E, ||.||_E)$. We give description of all surjective linear isometries acting in Banach symmetric ideals $(C_E, ||.||_{CE})$ in the case when $(C_E, ||.||_{CE})$ is a separable or a perfect Banach symmetric ideal and $C_E \neq C_2$.

## OSCILLATORY INTEGRALS AND WEIERSTRASS POLYNOMIALS

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The well-known Weierstrass theorem states that if $f(z, w)$ is holomorphic at a point $(z^0, w^0) \in \mathbb{C}_z^n \times \mathbb{C}_w$ and $f(z^0, w^0) = 0$, but $f(z^0, w) \neq 0$, then in some neighborhood $U = V \times W$of this point $f$ is represented as

$$f(z, w) = \left[ (w - w^0)^m + c_{m-1}(z)(w - w^0)^{m-1} + ... + c_0(z) \right] \varphi(z, w),$$

*where* $c_k(z)$ *are holomorphic in* $V$ *and* $\varphi(z,w)$ *is holomorphic in* $U$, $\varphi(z,w) \neq 0$, $(z,w) \in U$.

In recent years, the Weierstrass representation has found a number of applications in the theory of oscillatory integrals. Using a version of Weierstrass representation the first author obtain a solution of famous Sogge -Stein problem. He obtained also close to a sharp bound for maximal operators associated to analytic hypersurfaces.

In the obtained results the phase function is an analytic function at a fixed critical point without requiring a condition $f\left(z^0,w\right) \neq 0$. It is natural to expect the validity of Weierstrass theorem without requiring a condition $f\left(z^0,w\right) \neq 0$, in form

$$f(z,w) = \left[c_m(z)\left(w-w^0\right)^m + c_{m-1}(z)\left(w-w^0\right)^{m-1} + ... + c_0(z)\right]\varphi(z,w).$$

Such kind of results may be useful to studying of the oscillatory integrals and in estimates for maximal operators on a Lebesgue spaces. However, the well-known Osgood counterexample shows that when $n > 1$ it is not always possible. We will show, that there is a global option, a global multidimensional (in $w$) analogue of this theorem is proved without the condition $f\left(z^0,w\right) \neq 0$.

## (THERMO) DYNAMIC SYSTEMS IN BIOLOGY AND PHYSICS

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We define several biological and physical dynamical systems and give their properties depending on time and temperature. Moreover we discuss some open problems.

# MATHEMATICS

# Short Communications

## ISOMETRIES OF $F$-SPACES OF log-INTEGRABLE MEASURABLE FUNCTIONS

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A well-known result of J.Lamperti [3] gives a complete description of all surjective linear isometries $U : L_p(\Omega_1, \mathcal{A}_1, \mu_1) \to L_p(\Omega_2, \mathcal{A}_2, \mu_2)$ for the $L_p$-spaces $L_p(\Omega_i, \mathcal{A}_i, \mu_i)$, where $1 \le p < \infty,\ p \ne 2,$ and $(\Omega_i, \mathcal{A}_i, \mu_i)$ are an arbitrary measure spaces with a finite measures $\mu_i,\ i = 1, 2$.

An important metrizable analogue of $L_p$-space is an $F$-space $L_{\log}(\Omega, \mathcal{A}, \mu)$ of all log-integrable measurable functions, introduced in [2]. The $F$-space $L_{\log}(\Omega, \mathcal{A}, \mu)$ is defined by the equality $L_{\log}(\Omega, \mathcal{A}, \mu) = \{f \in L_0(\Omega, \mathcal{A}, \mu) : \|f\|_{\log,\mu} = \int\limits_{\Omega} \log(1 + |f|)d\mu < +\infty\}$, where $L_0(\Omega, \mathcal{A}, \mu)$ is the algebra of all measurable functions defined on $(\Omega, \mathcal{A}, \mu)$ (equal almost everywhere functions are identified). It is known that $L_{log}(\Omega, \mathcal{A}, \mu)$ is a subalgebra in algebra $L_0(\Omega, \mathcal{A}, \mu)$. In addition, $F$-norm $\|\cdot\|_{\log,\mu}$ is defined special metric $\rho_{log,\mu}(f, g) = \|f - g\|_{\log,\mu},\ f, g \in L_{\log}(\Omega, \mathcal{A}, \mu)$, with respect to which the pair $(L_{log}(\Omega, \mathcal{A}, \mu), \rho_{log})$ is a complete metric topological vector space.

Let $(\Omega, \mathcal{A}, \mu)$ (respectively, $(\Lambda, \mathcal{B}, \nu)$) be an arbitrary measure space with a finite measure. Denote by $\nabla_\mu$ (respectively, $\nabla_\nu$) the complete Boolean algebra of all equivalence classes $[A],\ A \in \mathcal{A}$ (respectively, $[B],\ B \in \mathcal{B}$). It is known that $\widehat{\mu}([A]) = \mu(A)$ (respectively, $\widehat{\nu}([B]) = \nu(B)$) is a strictly positive countably additive finite measure on $\nabla_\mu$ (respectively, on $\nabla_\nu$), which we also denoted by $\mu$ (respectively, $\nu$). Put $L_0(\nabla_\mu) = L_0(\Omega, \mathcal{A}, \mu)$ (respectively, $L_0(\nabla_\nu) = L_0(\Lambda, \mathcal{B}, \nu)$) and consider in algebra $L_0(\nabla_\mu)$ the following subalgebra $L_{\log}(\nabla_\mu) = \{f \in L_0(\nabla_\mu) : \|f\|_{\log,\mu} = \int\limits_{\Omega} \log(1 + |f|)d\mu < +\infty\}$.

We recall that a linear bijection $\Phi : L_0(\nabla_\mu) \to L_0(\nabla_\nu)$ is called isomorphism if $\Phi(fg) = \Phi(f)\Phi(g)$ for all $f, g \in L_0(\nabla_\mu)$. If an isomorphism $\Phi : L_0(\nabla_\mu) \to L_0(\nabla_\nu)$ preserves measures $\mu$ and $\nu$, i.e. $\nu(\Phi([A]) = \mu([A])$ for all $[A] \in \nabla_\mu$, then by [1, Proposition 3] we get $\Phi(L_{\log}(\nabla_\mu)) = L_{\log}(\nabla_\nu)$, in addition, $\Phi : (L_{\log}(\nabla_\mu), \|\cdot\|_{\log,\mu}) \to (L_{\log}(\nabla_\nu), \|\cdot\|_{\log,\nu})$ is the surjective linear isometry. The following theorem shows that there are not other surjective linear isometries from $(L_{\log}(\nabla_\mu), \|\cdot\|_{\log,\mu})$ onto $(L_{\log}(\nabla_\nu), \|\cdot\|_{\log,\nu})$.

**Theorem.** *Let $U : (L_{\log}(\nabla_\mu), \|\cdot\|_{\log,\mu}) \to (L_{\log}(\nabla_\nu), \|\cdot\|_{\log,\nu})$ be a surjective linear isometry. Then there exists uniquely isomorphism $\Phi : L_0(\nabla_\mu) \to L_0(\nabla_\nu)$ such that $\nu(\Phi([A])) = \mu([A])$ for all $[A] \in \nabla_\mu$ and $U(f) = \Phi(f)$ for all $f \in L_{\log}(\nabla_\mu)$.*

**Corollary.** *Let $\mu$ and $\nu$ be a strictly positive countably additive finite measures on complete Boolean algebra $\nabla$, and let there is not preserves measures $\mu$ and $\nu$ isomorphism* *from $\nabla$ onto $\nabla$. Then there is not a surjective linear isometry from $(L_{\log}(\nabla_\mu), \|\cdot\|_{\log,\mu})$ onto $(L_{\log}(\nabla_\nu), \|\cdot\|_{\log,\nu})$.*

Note that for $L_p$-spaces $(L_p(\nabla_\mu), \|\cdot\|_p)$ and $(L_p(\nabla_\nu), \|\cdot\|_p)$, $1 \le p \le \infty$, there always exists a surjective linear isometry from $(L_p(\nabla_\mu), \|\cdot\|_p)$ onto $(L_p(\nabla_\nu), \|\cdot\|_p)$ [3].


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## EXTENSION OF RELATIVE-RISK POWER ESTIMATOR UNDER DEPENDENT RANDOM CENSORED DATA


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The aim of paper is considering the problem of estimation of conditional survival function in the case of right random censoring with presence of covariate. Let's consider the case when the support of covariate $C$ is the interval $[0, 1]$ and we describe our results on fixed design points $0 \le x_1 \le x_2 \le ... \le x_n \le 1$ at which we consider responses (survival or failure times) $X_1, ..., X_n$ and censoring times $Y_1, ..., Y_n$ of identical objects, which are under study. These responses are independent and nonnegative random variables (r.v.-s) with conditional distribution function (d.f.) at $x_i$,$F_{x_i}(t) = P(X_i \le t/C_i = x_i)$. They are subjected to random right censoring, that is for $X_i$ there is a censoring variable $Y_i$ with conditional d.f. $G_{x_i}(t) = P(Y_i \le t/C_i = x_i)$ and at $n-$th stage of experiment the observed data is $S^{(n)} = \{(Z_i, \delta_i, C_i), 1 \le i \le n\}$, where $Z_i = min(X_i, Y_i), \delta_i = I(X_i \le Y_i)$ with $I(A)$ denoting the indicator of event $A$. Note that in sample $S^{(n)}$ r.v. $X_i$ is observed only when $\delta_i = 1$. Commonly, in survival analysis to assume independence between the r.v.-s $X_i$ and $Y_i$ conditional on the covariate $C_i$. But, in some practical situations, this assumption does not hold. Therefore, in this article we consider a dependence model in which dependence structure is described through copula function. So let $S_x(t_1, t_2) = P(X_x > t_1, Y_x > t_2)$, $t_1, t_2 \ge 0$, the joint survival function of the response $X_x$ and the censoring variable $Y_x$ at $x$. Then the marginal survival functions are ${S_x}^X(t) = 1 - F_x(t) = S_x(t, 0)$ and ${S_x}^Y(t) = 1 - G_x(t) = S_x(0, t), t \le 0$. We suppose that the marginal d.f.-s $F_x$ and $G_x$ are continuous. and the joint survival function $S_x(t_1, t_2)$ can be expressed as

$$S_x(t_1, t_2) = C_x(S_x^X(t_1), S_x^X(t_2)) \, , \; t_1, t_2 \ge 0,$$

where $C_x(u, v)$ is a known Archimedean copula function depending on $x, S_x^X$ and $S_x^Y$ in a general way. We consider estimator of d.f. $F_x$ which is extension of relative-risk power estimator proposed in [1,2].


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## COMPARISON OF THE EFFICIENCY OF MOLECULAR DESCRIPTORS IN MODELING THE STRUCTURE-ACTIVITY RELATIONSHIP


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**Introduction**

To date, there are two main approaches in drug design based on ligand representation: a) a search for active compounds based on chemical similarity and b) predictions based on of the structure-activity relationship (QSAR) of various datasets. Approaches based on similarity require one known hit compound, while QSAR models can be developed with a sufficiently large data set of biologically active compounds. The purpose of this work is to test the use of various sets of descriptors by the kNN-QSAR method, including descriptors generated by Rcdk, Dragon and SPCI software, which allowed interpreting the results of modeling for a set of 90 nitroaromatic compounds; an example of the interpretation of the simulation result is shown.

**Material and methods**

In this study, a set of 90 nitroaromatic compounds taken from [1] was analyzed. For computational experiments, the data were presented in the .SDF format and standardized by the Chemaxon program. We used the kNN-QSAR method [2] and the approach to structural interpretation.

**Results and discussion**

The first computational experiment was aimed at exploring different sets of descriptors generated by the Rcdk package, from which the Simulated Annealing procedure selects different sets of descriptors for which the regression models are built. On the selected sets of descriptors, about 10 models were obtained, of which 5, according to the criteria of kNN-QSAR, can be considered acceptable. In the second computational experiment, other descriptor generation systems, Dragon 6.0 ,SPCI were used. The third computational experiment performed on Sirms descriptors, generated by SPCI. The original 90 compounds in the SDF format and their activities in the form Log (1/C) were loaded into the SPCI program, in order to obtain a structural interpretation.

**Conclusion**

Thus, this study once again confirmed the need to select the appropriate system of descriptors in each specific case, which was repeatedly emphasized by other authors. In addition, the work shows an example of the interpretation of the constructed models.

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## 2-LOCAL ISOMETRIES OF COMPLEX FUNCTION LORENTZ SPACES


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The study of linear isometries on classical Banach spaces was initiated by S. Banach. He described all isometries on the space $L_p[0,1]$ with $p \neq 2$. J.Lamperti characterized all linear isometries on the $L_p$-space $L_p(\Omega, \mathcal{A}, \mu)$, where $(\Omega, \mathcal{A}, \mu)$ is a measurable space with a complete $\sigma$-finite measure $\mu$. M.G.Zaidenberg gave a description of all surjective linear isometries on the complex symmetric space $E(\Omega, \mathcal{A}, \mu)$, where $\mu$ is a continuous measure. Here we will establish the explicit form of all surjective 2-local isometries acting in complex function Lorentz spaces.

Let $(\Omega, \mathcal{A}, \mu)$ be a measure space with a complete finite measure $\mu$, and let $L^0(\Omega)$ be an $*$-algebra of equivalence classes of all complex valued measurable functions on $(\Omega, \mathcal{A}, \mu)$. The non-increasing rearrangement $f^* : (0, \infty) \to (0, \infty)$ of $f \in L^0(\Omega)$ is defined by $f^*(t) = \inf\{\lambda > 0 : \mu(\{|f| > \lambda\}) \leq t\},\ t > 0$, [1, Chapter 2, Definition 1.5].

Let $\psi$ be a concave function on $[0, \infty)$ with $\psi(0) = 0$ and $\psi(t) > 0$ for all $t > 0$, and let

$$\Lambda_\psi(\Omega, \mathcal{A}, \mu) = \left\{ f \in L^0(\Omega) : \ \|f\|_\psi = \int_0^\infty f^*(t) d\psi(t) < \infty \right\}$$

be the corresponding *Lorentz space*. In the paper [2] it was shown that continuous linear surjection $U : \Lambda_\psi(\Omega, \mathcal{A}, \mu) \to \Lambda_\psi(\Omega, \mathcal{A}, \mu)$ is an isometry if and only if there exist uniquely function $h \in L^\infty(\Omega)$, $|h| \equiv 1$, and an isomorphism $\Phi : L^0(\Omega) \to L^0(\Omega)$ such that $\int\limits_\Omega \Phi(f) d\mu = \int\limits_\Omega f d\mu$ and $U(f) = h \cdot \Phi(f)$ for all $f \in \Lambda_\psi(\Omega, \mathcal{A}, \mu)$.

A surjective (not necessarily linear) mapping $T : \Lambda_\psi(\Omega, \mathcal{A}, \mu) \to \Lambda_\psi(\Omega, \mathcal{A}, \mu)$ is called *a surjective 2-local isometry*, if for any $f, g \in \Lambda_\psi(\Omega, \mathcal{A}, \mu)$ there exists a surjective linear isometry $U_{f,g}$ on $\Lambda_\psi(\Omega,\mathcal{A},\mu)$ such that $T(f)=U_{f,g}(f)$ and $T(g)=U_{f,g}(g)$. It is clear that every surjective linear isometry on $\Lambda_\psi(\Omega,\mathcal{A},\mu)$ is automatically a surjective 2-local isometry on $\Lambda_\psi(\Omega,\mathcal{A},\mu)$. Using the equality $\|T(f)-T(g)\|_\psi=\|V_{f,g}(f)-V_{f,g}(g)\|_\psi=\|f-g\|_\psi,\ f,g\in\Lambda_\psi(\Omega,\mathcal{A},\mu)$, and Mazur-Ulam Theorem [3. Chapter I, §1.3, Theorem 1.3.5.], we get that in the case a Lorentz space of real measurable functions every surjective 2-local isometry on $\Lambda_\psi(\Omega,\mathcal{A},\mu)$ is a linear. For complex case, this fact is not obvious.

**Theorem.** *Let $\Lambda_\psi(\Omega,\mathcal{A},\mu)$ be a complex function Lorentz space, and let $T$ be a surjective 2-local isometry on $\Lambda_\psi(\Omega,\mathcal{A},\mu)$. Then $T$ is a linear isometry on $\Lambda_\psi(\Omega,\mathcal{A},\mu)$, in particular, there exist uniquely function $h\in L^\infty(\Omega)$, $|h|\equiv 1$, and an isomorphism $\Phi: L^0(\Omega)\to L^0(\Omega)$ such that $\int_\Omega \Phi(f)d\mu=\int_\Omega f d\mu$ and $U(f)=h\cdot\Phi(f)$ for all $f\in\Lambda_\psi(\Omega,\mathcal{A},\mu)$.*


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## ON THE PROBLEMS OF CONSTRUCTING A DISCRETE LYAPUNOV FUNCTION FOR HYPERBOLIC SYSTEMS


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The report proposes a systematic approach to the design and investigation of the adequacy of computational models for a mixed problem for symmetric $t$-hyperbolic systems with dissipative boundary conditions. We study the difference splitting scheme for the numerical calculation of stable solutions of a two-dimensional linear hyperbolic system with dissipative boundary conditions in the case of variable coefficients with lower terms. A discrete analogue of the Lyapunov function is constructed for the numerical value of stable solutions of a two-dimensional linear hyperbolic system with dissipative boundary conditions in the case of variable coefficients with lower terms. An a priori estimate is obtained for the discrete analogue of the Lyapunov function. The obtained a priori estimate allows us to assert the exponential stability of the numerical solution. A theorem on the exponential stability of the solution of a differential problem and a difference splitting scheme for linear hyperbolic systems in the corresponding norms will be proved. Hence, it gives us the opportunity to prove the convergence of the numerical solution.

# SOFTWARE, ALGORITHMS AND METHODS OF DATA ENCRYPTION BASED ON NATIONAL STANDARDS

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The article provides a brief description of the cryptography service provider software developed by the authors, which is designed to create encryption keys, private and public keys of electronic digital signature, create and confirm authenticity of digital signature, hashing, encrypting and simulating data using the algorithms described in State Standards of Uzbekistan (SSU). It can be used in telecommunications networks, public information systems, government corporate information systems by embedding into application applications that store, process and transmit information that does not contain information related to state secrets, as well as in the exchange of information and ensuring the legal significance of electronic documents. The cryptography service provider includes the following functional components: a dynamically loadable library that implements a biophysical random number sensor; dynamic library that implements cryptographic algorithms in accordance with the SSU; module supporting the work with external devices; installation module that provides the installation of cryptography service provider in the appropriate environment of operation (environment). CSP provides the creation of private and public EDS keys and encryption keys; creation and confirmation of authenticity of EDS according to the algorithms described in SSU; the formation of derived encryption keys used by data encryption algorithms described in SSU; work with key information stored on external media; hashing of memory areas and other data according to the algorithms described in SSU; encryption of memory areas and other data in accordance with the data encryption algorithms described in SSU. CSP provides support for identifiers of algorithms and parameters for the implementation of compatibility with third-party cryptographic providers in terms of the ability to work with public-key certificates issued by third-party registration centers, provided they use the cryptographic algorithms described in SSU. The cryptography service provider provides the ability to work with digital certificates of public keys, which are structured binary in ASN.1 format, conforming to ITU-T X.509 v.3 standard and IETF RFC 5280, RFC 3739 Recommendations. CSP provides work with external key carriers such as USB-flash, eToken Aladdin (eToken PRO 72K (JAVA)). As part of CSP, modules are provided that provide for calling cryptographic functions through the Microsoft CryptoAPI 2.0 interface when running under Microsoft operating systems. In accordance with the functional purpose, CSP provides the generation of public and private EDS keys, hashing keys, functional keys and encryption keys for use, respectively, in the algorithms described in SSU.

# ISOMETRIES OF NONCOMMUTATIVE ATOMIC SYMMETRIC SPACES

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Let $\tau$ be a faithful normal semifinite trace on semifinite von Neumann algebra $\mathcal{M}$, let $Z(\mathcal{M})$ be a center of the algebra $\mathcal{M}$, and let $L_0(\mathcal{M},\tau)$ be an $*$-algebra of all $\tau$-measurable operators affiliated with $\mathcal{M}$. For each $x \in L_0(\mathcal{M},\tau)$ we denote by $e_\lambda(x)$ the spectral projection corresponding to the interval $(\infty,\lambda]$, $\lambda \in \mathbb{R}$, where $\mathbb{R}$ is the field of real numbers.

Let $L_\tau(\mathcal{M})$ be the $*$-subalgebra in $L_0(\mathcal{M},\tau)$ of all $x$ such that $\tau(\mathbf{1} - e_\lambda(|x|)) < \infty$ for some $\lambda = \lambda(x) > 0$, where $\mathbf{1}$ is a unit in $\mathcal{M}$. If $x \in L_0(\mathcal{M},\tau)$ then a non-increasing rearrangement $\mu_t(x)$ of an operator $x$ is defined by the equality $\mu_t(x) = \inf\{\lambda > 0 : \tau(\mathbf{1}-e_\lambda) \leq t\}$ (see, for example, [2]).

A non-zero linear subspace $E \subset L_\tau(\mathcal{M})$ with a Banach norm $\|\cdot\|_E$ is called a symmetric space if the conditions $x \in E,\ y \in L_\tau(\mathcal{M}),\ \mu_t(y) \leq \mu_t(x),$ for all $t > 0$ imply that $y \in E$ and $\|y\|_E \leq \|x\|_E$ (see, for example, [3]).

It is known that $L_1(\mathcal{M},\tau)\cap\mathcal{M} \subset E \subset L_1(M,\tau)+\mathcal{M}$ for any symmetric space $E$, where $L_1(\mathcal{M},\tau) = \{x \in L_\tau(\mathcal{M}) : \|x\|_1 = \int\limits_0^\infty \mu_t(x)dt < \infty\}$.

A symmetric space $(E, \|\cdot\|_E)$ is said to have Fatou property if conditions $0 \leq x_n \in E$, $x_n \leq x_{n+1}$ and $\sup\limits_n \|x_n\|_E < \infty$ imply that there exists $x = \sup\limits_{n\geq 1} x_n \in E$ and $\|x\|_E = \sup\limits_{n\geq 1} \|x_n\|_E$.

We say that a bijective linear map $J\colon \mathcal{M} \to \mathcal{M}$ is Jordan automorphism if $J(x^2) = (J(x))^2$ for all $x \in \mathcal{M}$.

Recall that a linear operator $V\colon E \to E$ is called an isometry if $\|V(x)\|_E = \|x\|_E$ for all $x \in E$.

**Theorem 1.** *Let $E$ be a symmetric space with Fatou property, and let $V\colon E \to E$ be a surjective isometry. Then there exist an unique Jordan automorphism $J$ of the algebra $\mathcal{M}$, an unitary operator $u \in \mathcal{M}$ and a positive operator $h \in L_\tau(Z(\mathcal{M}))$ with support $s(h) = \mathbf{1}$ such that $V(x) = huJ(x)$ for all $x \in E \cap \mathcal{M}$.*

Note that in the case when an algebra $\mathcal{M}$ is an algebra $\mathcal{B}(\mathcal{H})$ of all bounded linear operators, acting in separable Hilbert space $H$, the statement of the theorem 1 was obtained in [1].

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# MATHEMATICAL MODELING OF SOLUTIONS LINEAR AND GEOMETRICALLY NONLINEAR PROBLEMS OF SPATIAL LOADING OF RODS

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At present stage of development of the mechanics of continuous media the interest to the problems of nonlinear theory of elasticity is explained by several reasons. Firstly, in practice, different bodies experience finite deformation, where materials exhibit substantial elastic properties. Their behavior is very different from the one predicted by linear theory. Proper accounting of non-linearity is particularly important in calculating the products made of elastomeric, plastic and other materials. Secondly, a number of phenomena, experimentally observed at certain strains (e.g., torsion) can not be described theoretically, retaining in the solution only linear terms relative to the gradient of displacement.

Thirdly, new materials, and non-linear behavior of known ones require the development of new mathematical models, which adequately describe their properties. Therefore, the solution (in the framework of nonlinear theory of elasticity) of the problems for certain basic experiments (tension, torsion, bending, etc.) using various determinant relationships allows us to check the suitability of the latter, to experimentally determine their characteristics at high strains, as well as to compare the behavior of various materials.

The building of the adequate mathematical models of these materials with full account of the nonlinearity should be based primarily on modeling of classic experiments, and, consequently, on the solution of fundamental problems of the theory of elasticity, describing a simple deformation of bodies (tension, torsion, bending, etc.). At the same time the solution of boundary value problems of the nonlinear theory of elasticity in most cases is difficult because the elastic potentials used present quite complex expressions, reducing to essentially nonlinear equations; their solution can't be found in analytical form.

At the same time, the majority of the studied nonlinear elastic potentials present a rather bulky expressions, which make analytical derivation of the boundary value problem of equilibrium even in the cases of simple loading an extremely time-consuming one and not always reliable. In addition, the change of the specific potential energy function often leads to the need to derive all the equations afresh. However, the process of derivation of boundary value problems of equilibrium is strictly algorithmic.

The aim of research is the development of mathematical models, computational algorithms and software tools for investigating the statics and dynamics of linear and geometrically nonlinear problems of stress-strain state of rods of complex configuration under spatial loading. The objects of research are strained processes in linear and geometrically nonlinear problems of the statics and dynamics of rods of complex configuration under spatial loading.

**STUDY OF PROPERTIES OF SOLUTIONS OF CROSS-DIFFUSION MODEL OF REACTION-DIFFUSION WITH DOUBLE NONLINEARITY**

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Consider in the domain $Q = \{(t,x) : 0 < t < \infty, x \in R^N\}$ the Cauchy problem for degenerate parabolic system with variable density

$$\begin{cases} \frac{\partial u_1}{\partial t} = \nabla \left( |x|^n \left| \nabla u_1^k \right|^{p-2} \nabla u_2^{m_1} \right) + k_1 \left( u_1 - u_1^{\beta_1} \right), \\ \frac{\partial u_2}{\partial t} = \nabla \left( |x|^n \left| \nabla u_2^k \right|^{p-2} \nabla u_1^{m_2} \right) + k_1 \left( u_2 - u_2^{\beta_2} \right), u_1 \mid_{t=0} = u_{10}(x)\ , u_2 \mid_{t=0} = u_{20}(x) \end{cases} \tag{1}$$

describing the problem of a biological population of Kolmogorov-Fisher type. Diffusion coefficients are equal$|x|^n \left|\nabla u_1^k\right|^{p-2}$,$|x|^n \left|\nabla u_2^k\right|^{p-2}$; numeric parameters $m_1, m_2, n, p, \beta_1, \beta_2$ - positive real numbers, $u_1 = u_1(t,x) \geq 0$, $u_2 = u_2(t,x) \geq 0$ - population density,$\nabla(.) = grad_x(.)$.

It is show have Fisher phenomes a finite saveel propagation in contrast prohem to the Kolmogorov Fisher model. In this work results of the numerical experiments disscused. The estimate and asymptotics of self-similar solutions of system (1) established.

Consider the nontations

$$\bar{f}(\xi)_1 = A(a - \xi^{p/(p-1)})_+{}^{n_1},\ \bar{f}(\xi)_2 = B(a - \xi^{p/(p-1)})_+{}^{n_2}$$

where $\xi = \varphi(|x|)/\tau^{1/p}$,$n_1 = \frac{(p-1)[k(p-2)-m_1]}{[k(p-2)]^2 - m_1 m_2}$,$n_2 = \frac{(p-1)[k(p-2)-m_2]}{[k(p-2)]^2 - m_1 m_2}$.

$\varphi(|x|) = |x|^{p_1}/p_1$, $p_1 = (p-n)/p$, $s = pN/(p-n)$,$p > n$

$\varphi(|x|) = \ln(|x|)$,$p = n$.

In particular, it takes place

**Theorem.** *Let* $u_i(0,x) \leq z_i(0,x),\ x \in R^N,\ \ i = 1,2$ *Then for the solution of problem (1) in the domain Q, takes place estimate* $u_1(t,x) \leq z_1(t,x) = (T+t)^{-\gamma_1}\bar{f}_1(\xi)$, $u_2(t,x) \leq z_2(t,x) = (T+t)^{-\gamma_2}\bar{f}_2(\xi)$,$\xi = \varphi(|x|)/\tau^{1/p}$, $\gamma_1 = -(\beta_1 - 1)^{-1}$, $\gamma_2 = -(\beta_2 - 1)^{-1}$.

The functions $\bar{f}_i(\xi)$ $(i = 1,2)$ are defined above.

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## BY THE ASYMPTOTICS OF THE SOLUTION OF THE HEAT CONDUCTION PROBLEM WITH DOUBLE NONLINEARITY WITH ABSORPTION AT A CRITICAL PARAMETER

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In this paper, we study the asymptotic behavior (for $t \to \infty$) of solutions of the Cauchy problem for a nonlinear parabolic equation with a double nonlinearity, describing the diffusion of heat with nonlinear heat absorption at the critical value of the parameter $\beta$.

In the domain $Q = \{(t, x) : t > 0, x \in R^N\}$ the following Cauchy problem

$$\frac{\partial u}{\partial t} = \nabla \left( |x|^n u^{m-1} \left| \nabla \mathrm{u^k} \right|^{p-2} \nabla \mathrm{u} \right) - u^{\beta_*} \quad (1)$$

$$u\,(0, x) = u_0\,(x) \geq 0,\ x \in R^N \qquad (2)$$

t and x are, respectively, the temporal and spatial coordinates where $m \geq 1$, $k \geq 1$, $p \geq 2$ given numerical parameters, characterizing the nonlinear medium. $\nabla(\cdot) = \underset{x}{grad}(\cdot)$ , $\beta_* = k(p-2) + m + \frac{p}{s}$ $s = p\frac{N}{p-n}$ , $n < p$ ($\beta_*$ -critical value, N- dimension).

The importance of studying the properties of solution for critical values of parameters is explained by the fact that in this case the asymptotics of the solution behaves differently. At critical values of the parameters, the asymptotics of the problem will undoubtedly change. For example, at critical values, infinite energy or localization may appear. A lot of works studied properties of solutions of problem with critical value of parameter $\beta$ and were established asymptotic behavior for $t \to \infty$ (see [2]-[6]). The long time asymptotic of solutions has been established for the critical value of parameter $\beta$ for problem (1)-(2) in case n=0,k=1,m=1,$\beta_* = p - 1 + p/N$ in [3],case p=2, $\beta_* = m + 2/N$ in [4] and case m=1,p=2, $\beta_* = 1 + 2/N$ in [2].

In this work the long time asymptotics of the solution to problem (1) - (2) for $m \geq 1$, $k \geq 1$, $p \geq 2$ with critical value of parameter $\beta_* = k(p-2) + m + \frac{p}{s}$, $s = p\frac{N}{p-n}$ , $n < p$ were established. The following asymptotics for long time

$$u(t, x) = (t \ln t)^{-\frac{1}{\beta_* - 1}} \bar{f}(\xi) \qquad (3)$$

where

$\bar{f}(\xi) = (a - b\xi^{\frac{p}{p-1}})_+^{\frac{p-1}{m+k(p-2)-1}}$, $\xi = \varphi\,(x)\,[\tau(t)]^{-1/p}, \varphi\,(x) = \frac{p}{p-n}|x|^{\frac{p-n}{p}}, p > n$, $\tau(t) \sim t^{\alpha_1} \ln^{\alpha_2} t,\ \ t \sim \infty$, $b = (m + k(p-2) - 1)/(k^{p-2} p^{\frac{p}{p-1}})$,$m + k(p-2) - 1 > 0$

$$\alpha_1 = \frac{k(p-2) + m - 1}{(p + (k(p-2) + m)N)p}, \alpha_2 = -\frac{k(p-2) + m - 1}{(p + (k(p-2) + m)N)p} + 1$$

For construction of solution (3) were used the method of standard equations [1]. Proofs of the proposal based on principle of comparison solutions. The resulting asymptotics of the solutions were used as an initial approximation for numerical computation.

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## CREATING A SMART MEDICAL SYSTEM BASED ON NEW TECHNOLOGIES

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The Resolution of the President of the Republic of Uzbekistan "On measures to further develop the national information communication system of the Republic of Uzbekistan" (PQ-1989) of 27 June 2013 and the Resolution of the Cabinet of Ministers of 31 December 2013 adopted in line with the Resolution of the Cabinet of Ministers of the Republic of Uzbekistan "On Measures for Implementation of the ICT Development System in the Republic of Uzbekistan"number 355 and timely, uninterrupted implementation of the requirements and requirements of PF 5349 of February 19, 2018 and the implementation of this program on the basis of creating a smart medical system is being made.

One of the basic requirements in the creation a smart medical system is a universally-built system that is flexible with other existing systems, can be created in modern open source software that does not require a license and ability to be combined with the large amount of data (BIG DATA), multimedia and other features.

Considering the development of new technologies in the software industry were selected next software as fore standing, PHP 7.1 and PHP framework Yii 2.0, MVP, and Postgres SQL 9.0 for databases. The main advantages of these technologies are:

- The width of choice of the Php developers: competition is that nowadays it is possible to find thousands of programmers all over the world and in Uzbekistan that can handle with this technology, because of the simplicity of the programming language of Php and the ease of later entering changes to the software.

- The intelligent system software is based on the microservice structure. The main advantage of this is that the entire system is based on the microservice and every single microservice works independently therefore, a microservice does not interfere with the operation of another part of the microservice, as a result of which the microservice in any department is not subject to the risk of system upgrades or interruptions in the work o interface, and the microservices in other sections continue to work in their own situation.

- PostgresSql was designed to work with the large amount of data (BIG DATA), which the medical database requires and indexes in the software were clearly released in working with all tables. This helps to reduce the system pressure by 40% to 80%

- Since PostgresSQL is licensed under the Postgres GNU license, it is absolutely free and can be easily modified.

- Since PHP + Postgres Sql are completely new, that appeared so many possibilities to develop for instance it is now possible to automate the work of the branches.

- No need for dedicated and valuable servers: With the advent of new technologies, many offers can now be seen in cloud computing. It is important to note that nowadays valuable servers are useless for the software. It can be evaluated for more than one times in data security.

- Cloud technologies provide secure inter-server communications avoiding problems of onsite server management.

## NUMERICAL MODELING WAVE TYPE STRUCTURES IN NONLINEAR DIFFUSION MEDIUM WITH DAMPING

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In this work in the domain $Q = \{(t,x) : t > 0, x \in R\}$ investigated the following diffusion equation with a damping term with convective transfer speed of which depends from time

$$\frac{\partial u}{\partial t} = \frac{\partial}{\partial x}\left(\left|\frac{\partial u^k}{\partial x}\right|^{p-2}\frac{\partial u^m}{\partial x}\right) - v(t)\frac{\partial u}{\partial x} - bu^{q_1}\left|\frac{\partial u^m}{\partial x}\right|^{p_1}, x > 0, t > 0 \qquad v(t). \qquad (1)$$

When $v(t) = 0, m = 0,\ p = 2$ (1) is the well-known Hamilton-Jacobi [1] equation. In the case, $k = m$ (1) is the well-known evolutionary p-Laplacian equation with the absorption term. When $p = 2$ (1) is porous media equation (PME) with the absorption term. These equations come from many fields such as physics, fluid mechanics, biological population et al (see [1] and the literature therein) Various qualitative properties of the solution of the problem (1) and nonlinear phenomena for different particular value of the numerical parameters and when $v(t) = 0$ intensively studied by many authors (see [1-4] and references therein). Huashui Zhan [1] applying the standard iteration method in the case a sufficient condition gave to the existing of the singular self-similar solutions of the equation (1) and a classification of these singular solutions.

In this wok we study wave type solution

$$u(t,x) = f(\xi),\ \xi = ct\int_0^t v(y)dy \pm x$$

of the equation (1). The exact weak wave type solution for some value of the numerical parameters in one dimensional and multidimensional cases founded. Due to a convective transfer speed of which depends from a time the space localized phenomenon established. Computer modeling of the wave type solution based an exact solution carried out. Asymptotic behavior of a wave type solution near the free boundary established. Results of the numerical computations presented in visualization form discussed.

Wave type equation of the (1) satisfy to the following degenerate type differential equation

$$c\frac{df}{d\xi} = \frac{d}{d\xi}\left(\left|\frac{df^k}{d\xi}\right|^{p-2}\frac{df^m}{d\xi}\right) - bf^{q_1}\left|\frac{df^m}{d\xi}\right|^{p_1}, \tag{2}$$

because degeneracy of the equation we consider weak solution having physical sense. We prove that function

$$f(\xi) = A\xi^\lambda, \lambda = \frac{p-1}{k(p-2)+m} \tag{3}$$

are exact solution of (2) if the constant A satisfy to the following algebraic equation

$$(\lambda k)^{p-2}A^{k(p-2)+m} - b\lambda^{p_1-2}A^{mp_1+q_1-1} = c. \tag{4}$$

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# TO PROPERTIES OF SOLUTIONS OF THE CAUCHY PROBLEM FOR A DEGENERATE NONLINEAR CROSS SYSTEMS WITH VARIABLE DENSITY AND ABSORPTION

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This talk devotes to the Cauchy problem for the following degenerate nonlinear cross systems with variable density and absorption:

$$\begin{array}{l} L_1(u,v) = |x|^l \frac{\partial u}{\partial t} + div\left(|x|^n v^{m_1-1}\left|\nabla u^k\right|^{p-2}\nabla u\right) - |x|^l \gamma_1(t)u = 0, \\ L_2(u,v) = |x|^l \frac{\partial v}{\partial t} + div\left(|x|^n u^{m_2-1}\left|\nabla v^k\right|^{p-2}\nabla v\right) - |x|^l \gamma_2(t)v = 0, \end{array} \qquad (1)$$

$$u\left(0,x\right) = u_0\left(x\right) \geq 0, v\left(0,x\right) = v_0\left(x\right) \geq 0,\ x \in R^N \qquad (2)$$

in the domain $Q = \left\{(t,x) : t > 0, x \in R^N\right\}$ investigated where $k \geq 1,\ p, n, l, m_i\,,\ i = 1,2$ - given positive numbers, $\nabla(.) - \underset{x}{grad}(.)$, the functions $u_0\left(x\right),\ \ v_0\left(x\right) \geq 0,\ x \in R^N, 0 < \gamma_i(\mathrm{t}) \in C(0,\infty),\ i = 1,2.$

The system (1) describe set of physical process, for example process of mutual reaction - diffusions, heat conductivity, a polytrophic filtration of a liquid and gas in the nonlinear environment which capacity equally $\gamma_1(t)u, \gamma_2(t)v$. Different particular cases of the problem (1)-(2) considered in many works (for instance see [1-4] and references therein). The system (1) in the domain, where $u = v = 0$ is degenerated, and in the domain of degeneration it may have not the classical solution. Therefore, we study the weak solutions of the system (1) having physical sense [1-5]: The phenomenon of a finite speed propagation of disturbance, a space localization of solution for the cases $p \geq l + n$ Estimate of a weak solution from class:

$$0 \leqslant u, v,\ |x|^n v^{m_1-1}\left|\nabla u^k\right|^{p-2}\nabla u,\ \ |x|^n u^{m_2-1}\left|\nabla v^k\right|^{p-2}\nabla v \in C(Q)$$

to satisfying system (1), (2) in distribution sense[1]. The case $p > n + l$ and singular case $p = n$+l are considered.

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## TO ASYMPTOTIC OF WKB SOLUTION OF ONE DEGENERATE NONLINEAR SYSTEM

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Consider singular differential parabolic systems of the type

$$\begin{aligned} \frac{\partial u}{\partial t} &= \mathtt{div}\left(q(|x|)|\mathtt{grad}u|^{\alpha-1}\mathtt{grad}u\right) - p(|x|)u^{\beta_1}v^{-\lambda}, \\ \frac{\partial v}{\partial t} &= \mathtt{div}\left(q(|x|)|\mathtt{grad}v|^{\alpha-1}\mathtt{grad}v\right) - p(|x|)u^{-\mu}v^{\beta_2}, \end{aligned} \tag{1}$$

in an interval $[a, \infty)$, where $\alpha$, $\beta_1$, $\beta_2$, $\lambda$, $\mu$ are positive constants and $p$, $q$, are positive continuous functions on $[a, \infty)$. This type system describes processes of heat conductivity, nonlinear diffusion, process of filtration in liquid and gas in variable density [2-5] . A positive decreasing solution of (1) is called proper or singular according to whether it exists on $[a, \infty)$ or it cases to exist at a finite point of $(a, \infty)$. Then, conditions are established for the existence of proper solutions of (1) which are classified into moderately decreasing solutions and strongly decreasing solutions according to the rate of their decay as $x \to \infty$. Different qualitative properties of solution of initial value problem for particular value of numerical parameters and coefficients studied by many authors (see [1] and literature therein). Properties of positive solution stationary system (1) for particular value of parameters established in [2].

In this work we investigate asymptotic of compactly supported, regular, unbounded properties solution of the system using WKB solution [3]. The estimate of solution of stationary system established.

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**ON THE BIOLOGICAL POPULATION WITH DOUBLE NONLINEAR DIFFUSION OF A NON-DIVERGENT TYPE**

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Consider in the domain $Q = \{(t,x) : t > 0, x \in R\}$ the problem of the biological population of the reaction diffusion described by double nonlinear parabolic equation in non-divergent form

$$\frac{\partial u}{\partial t} = u^n(\nabla D_0 u^{m-1}|\nabla u|^{p-2}\nabla u) + b(t)u - c(t)u^\beta \tag{1}$$

with initial condition

$$u_{t=0} = u_0(x),\ x \in R, \tag{2}$$

where $m \geq 1$, $n \neq 0, 1$ $p \geq 2$, $\beta \geq 1$ given numerical parameters, $x \in R^N$; $0 \leq u_0(x) \leq 1$, $\nabla(\cdot) = \underset{x}{grad}(\cdot)$. Equation (1) is a generalization of the simplest diffusion model for a logistic model of population growth [1-2] of the type Malthus $b(t) = b,\ c(t) = 0,\ \beta = 1$, Fohrhulst $b(t) = b,\ c(t) = 0,\ \beta = 1$, and Ollie type $(b(t) = b, c(t) = c)$, $\beta = 2$ for the case of double nonlinear diffusion of a non-divergent type. Note that the considered equation is the best combination of the nonlinear slow diffusion equation $(n = 0, m + p - 3 > 0$, fast diffusion $n = 0,\ m + p - 3 < 0$ the $p-$ Laplace equation $(n = 0,\ m = 1)$ with lower terms. The case $n + m + p - 3 = 0$ is called a special case.

Since equation (1) in the region where $u = 0,\ \nabla u = 0$ is degenerate, therefore it may not have a classical solution in the domain of degeneracy [3]. Therefore, we consider weak solution of equation (1), having the property: $0 \leq u,\ u^n(\nabla D_0 u^{m-1}|\nabla u|^{p-2}\nabla u) \in C(Q)$ and satisfying to equation (1) in the distrubation sense [3].

As is known, for problem (1), (2) because the degeneration of the equation, the presence of a property of a finite velocity, the propagation of perturbations, is characteristic, that is, there exists a continuous function $l(t)$ such that when $u(t,x) \equiv 0$ at $|x| \geq l(t)$. The surface $|x| = l(t)$ is called the free bounary or front. In the case when $l(t) < \infty$ at $t > 0$, the solution is called spatially localized.

In this paper by construction the exact solution of the problem (1) for sufficiently arbitrary functions $b(t)$, $c(t)$, $u_0(x)$ we prove arising the following nonlinear effects: inertia effect of finite speed of propagation of population disturbances, the spatial localization of the flashes and the effect of finite lifetime of the population structure. The estimate of a weak solution established. The results of numerical experiments discussed. Exact solution consist results other authors [2-6] for the case $c(t) = b(t) = 0$, $n = 0$, $m = 1$ or $p = 2$.

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## 2-LOCAL DERIVATIONS OF POLYNOMIAL WITT ALGEBRAS

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In 1997, Šemrl [5] introduced 2-local derivations and 2-local automorphisms on algebras. A map $\Delta : \mathcal{L} \to \mathcal{L}$ (not necessarily linear) on an algebra $\mathcal{L}$ is called a *2-local derivation* if, for every $x, y \in \mathcal{L}$, there exists a derivation $D_{x,y} : \mathcal{L} \to \mathcal{L}$ such that $D_{x,y}(x) = \Delta(x)$ and $D_{x,y}(y) = \Delta(y)$. For a given algebra $\mathcal{L}$, the main problem concerning these notions is to prove that they automatically become a derivation (respectively, an automorphism) or to give examples of local and 2-local derivations or automorphisms of $\mathcal{L}$, which are not derivations or automorphisms, respectively. Solution of such problems for finite-dimensional Lie algebras over algebraically closed field of zero characteristic were obtained in [1],[2],[3] and [4]. Namely, in [3] it is proved that every 2-local derivation on a semi-simple Lie algebra $\mathcal{L}$ is a derivation and that each finite-dimensional nilpotent Lie algebra with dimension larger than two admits 2-local derivation which is not a derivation. In [1] the authors have proved that every local derivation on semi-simple Lie algebras is a derivation and gave examples of nilpotent finite-dimensional Lie algebras with local derivations which are not derivations. Concerning 2-local automorphism, Chen and Wang in [4] prove that if $\mathcal{L}$ is a simple Lie algebra of type $A_l, D_l$ or $E_k, k = 6, 7, 8)$ over an algebraically closed field of characteristic zero, then every 2-local automorphism of $\mathcal{L}$, is an automorphism. Finally, in [2] Ayupov and Kudaybergenov generalized this result of [4] and proved that every 2-local automorphism of a finite-dimensional semi-simple Lie algebra over an algebraically closed field of characteristic zero is an automorphism. Moreover, they show also that every nilpotent Lie algebra with finite dimension larger than two admits 2-local automorphisms which is not an automorphism.

In the present paper we study 2-local derivations of one-sided Witt algebra.

The classical one-sided Witt algebra $W_1$ is defined with basis $\{e_i |\ i \in \mathbb{Z},\ i \geq -1\}$ and brackets

$$[e_i, e_j] = (j - i)e_{i+j}, \text{ for all } \ i, j \geq -1,\ i, j \in \mathbb{Z}.$$

Thus by realizing $e_i$ as $x^{i+1}\frac{d}{dx}$, one immediately observes that $W_1 = Der\mathbb{C}[x] = \mathbb{C}[x]\frac{d}{dx}$ is the derivation algebra of the polynomial algebra $\mathbb{C}[x]$. Any derivation of the classical one-sided Witt algebra $W_1$ is an inner derivation (see[6]).

The following theorem is the main result of this note.

**Theorem.** Any 2-local derivation on $W_1$ is a derivation.


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## THE SOLUTION OF A CAUCHY PROBLEM FOR THE PFAFF EQUATION WITH CONTINUOUS COEFFICIENTS


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One of the classical types of differential equations is

$$\omega = a_0\left(X\right)dX_0 + a_1\left(X\right)dX_1 + a_2\left(X\right)dX_2 = 0, \tag{1}$$

called a Pfaff equation (where $X = (X_0, X_1, X_2) \in D$, $D \subset \mathbb{R}^{n+1}$).

Equation (1) defines a field of hyperplanes that is not always integrable, i.e. there may not exist a hypersurface tangent in each its point to the hyperplane of the field (1) [1]. If coefficients $a_i\left(X\right), i = 0, 1, 2$, continuously differentiable in $D$, then necessary and sufficient condition of integrability of (1) will be given by Frobenius's criterion [1], [2],[3],[4]:

$$\omega \wedge d\omega = 0. \tag{2}$$

Under the condition (2) there is a single integral surface passing through each point $X^0$, $X^0 \in D$, if the last not singular i.e. $a_i\left(X^0\right) \neq 0$ at least for one $i = 0, 1, 2$. We emphasize that the Frobenius criterion is a condition of integrability in a whole, i.e. concerns all points of the domain $D$.

For a definiteness assume $a_0(X) \neq 0$ in some neighbourhood of a point $X^0$, that will be denote by $D$ again. Then the equation (1) becomes equivalent to the system

$$\frac{\partial u}{\partial x_1} = f_1(x, u), \frac{\partial u}{\partial x_2} = f_2(x, u), \tag{3}$$

where $u = X_0$, $x = (x_1, x_2)$, $x_j = X_j$, $f_j(x, u) = -a_j(X)/a_0(X)$, $f = (f_1, f_2)$, $j = 1, 2$, $X^0 = (x_0, u_0)$.

It is clear the formulation of Frobenius integrability condition in the form (2) is meaningless without an assumption of differentiability of functions $f_j$, $j = 1, 2$. Therefore, first of all we should reformulate the criterion of integrability in the convenient form. To avoid unwieldy expressions, further we constraint ourselves with consideration of the case $n = 2$, re-denoting the variables $x_1$ and $x_2$ as $x$, $y$ respectively. Then the system (3) will take the following form

$$\frac{\partial u}{\partial x} = f(x, y, u), \tag{4}$$

$$\frac{\partial u}{\partial y} = g(x, y, u). \tag{5}$$

Thus we consider a Cauchy problem for the system (4)-(5). Suppose $f$ and $g$ are defined in a domain $D$, $D \subset \mathbb{R}^3$ and $(x_0, y_0, u_0) \in D$. The task is to find a solution $u(x, y)$ from the class $C^1$, satisfying the system (4)-(5) and the condition $u(x_0, y_0) = u_0$.

**Theorem 1.** *Let $f$ and $g$ are continuous functions in the domain $D$. Then the following propositions are equivalent:*

*a) the Cauchy problem for the system (4)-(5) is solvable at any $(x_0, y_0, u_0) \in D$;*

*b) for any point $(x_0, y_0, u_0) \in D$ integral equations*

$$v(x, y) = u_0 + \int_{x_0}^{x} f[s, y, v(s, y)]\, ds + \int_{y_0}^{y} g[x_0, t, v(x_0, t)]\, dt \tag{2}$$

$$w(x, y) = u_0 + \int_{x_0}^{x} f[s, y_0, w(s, y_0)]\, ds + \int_{y_0}^{y} g[x, t, w(x, t)]\, dt \tag{3}$$

*have coinciding solutions.*

**Corollary.** *If the functions $f$ and $g$ are continuously differentiable, then the Frobenius condition is equivalent to the statement b) of Theorem 1.*

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## UNBOUNDED LIMIT SETS OF DYNAMICAL SYSTEMS

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Consider a dynamical system

$$\dot{\boldsymbol{z}} = \boldsymbol{f}(\boldsymbol{z}), \tag{1}$$

$\boldsymbol{z} \in \mathbb{R}^d$, $d \geq 2$. Further, we will deal with a fixed trajectory $\boldsymbol{z}(t)$, passing through a given initial point $\xi$, $\boldsymbol{z}(0) = \xi$ and its $\omega$-limit set $\Omega$. The case of $\Omega \neq \varnothing$ will be interested only.

Well known $\Omega$ is always closed invariant set. If $\Omega$ is bounded, then it is non-empty connected compact set [1-3]. $\Omega$ can be a point or closed trajectory in the simplest cases but in the general case $\Omega$ may have rather complicated structure especially for $d \geq 3$. The structure of $\Omega$ was studied rigorously in the case of $d = 2$ [4,5].

According to the property mentioned above, a disconnected $\omega$-limit set is always unbounded. Of course this statement does not exclude possibility that $\Omega$ can have a bounded connectivity component admitting unboundedness of some other components or even a case when all components are bounded, but their union is unbounded. Therefore the following statement clarifies the property, mentioned above.

**Theorem 1.** *If the $\omega$-limit set of the system () is not connected, then every connectivity component is unbounded.*

It is also studied properties a number of connectivity components of $\Omega$ when it is unbounded.

**Theorem 2.** *Let $\Omega$ be the disconnected $\omega$-limit set of a trajectory $\boldsymbol{z}(t)$ of a dynamical system (1). If each connectivity component of the set $\Omega$ contains at least one non-equilibrium point, then the number of components of $\Omega$ is not more than countable.*

**Corollary.** *If each connectivity component of $\Omega$, with the exception of no more than a countable number of them contains a non-equilibrium point, then the number of connected components is not more than countable.*

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## ON A GENERALIZATION OF THE THEOREM COMPARISON FOR QUASILINEAR PARABOLIC EQUATIONS

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There are studied some properties of solutions of displaced problems for a quasi linear parabolic equation in this work [2].

$$u_t - \sum_{i,j=1}^{n} A_{ij}(x,t,u,u_x)u_{x_i x_j} = F(x,t,u,u_x) \tag{1}$$

in the cylindrical region $Q_T = G_x\,(0,T]\,u \in R^{n+1}$ The problem is reduced to the study of the solution of the initial problem for the equation

$$\frac{du}{dt} = F_1(t,u). \tag{2}$$

The theorem is proved for (1) under general boundary conditions. The following result is generalized for (1). Let and continuous in $(0,T]$,differentiable in $(0,T]$ and

$$\left\{ \frac{du}{dt} \le g(u(t),t)\frac{dv}{dt} \ge g(v(t),t) \right. \tag{3}$$

Suppose, that $g(u,t) - g(v,t) \le \Phi(u-v,t)$ for $|u|\,,|v| < M$ and $0 < t \le T$ where each $M > 0, \Phi \in U$. Then if $u(0) \le v(0)$ then $u(t) \le v(t)$ for $0 \le t \le T$.

**Theorem.** *Let $u(t,x)$ both $v(t,x)$ functions from $C^{1,2}(Q_T)$ and satisfies*

$$u_t - \sum_{i,j=1}^{n} A_{ij}(t,x,u,u_x)u_{x_i x_j} \le B(t,x,u,u_x)$$

$$v_t - \sum_{i,j=1}^{n} A_{ij}(t,x,v,v_x)v_{x_i x_j} \le B(t,x,v,v_x)$$

*$u(0,x) \le v(0,x)$ for all$x \in \partial G$,and for everyone $t \in (0,T]$ at each point $x \in \partial G$ we will have $u(t,x) \le v(t,x)$ or $u_\xi(t,x) \le v_\xi(t,x)$, where is $\xi = \xi(t,x)$ the external field of directions, besides we assume that, and it is more limited in $Q_T$ $u(t,x)$ or $v(t,x)$ has bounded first and second order derivatives in $\overline{Q_T}$ c [1] then $u(t,x) \le v(t,x)$ in $\overline{Q_T}$.*

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## MEAN ERGODIC THEOREM IN NONCOMMUTATIVE ATOMIC SYMMETRIC SPACES

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Let $\mathcal{M}$ be a semifinite von Neumann algebra with a unit $\mathbf{1}$, let $\tau$ be a faithful normal semifinite trace on $\mathcal{M}$, and let $L_0(\mathcal{M},\tau)$ be an $*$-algebra of all $\tau$-measurable operators affiliated with $\mathcal{M}$. For each $x \in L_0(\mathcal{M},\tau)$ we denote by $e_\lambda(x) = \{x \le \lambda\}$ the spectral projector corresponding to the interval $(\infty, \lambda]$, $\lambda \in \mathbb{R}$, where $\mathbb{R}$ is the field of real numbers.

Let $L_\tau(\mathcal{M})$ be the $*$-subalgebra in $L_0(\mathcal{M},\tau)$ of all operators $x \in L_0(\mathcal{M},\tau)$ such that $\tau(\mathbf{1} - e_\lambda(|x|)) < \infty$ for some $\lambda = \lambda(x) > 0$. If $x \in L_0(\mathcal{M},\tau)$ then a non-increasing rearrangement $\mu_t(x)$ of an operator $x$ is defined by the equality $\mu_t(x) = \inf\{\lambda > 0 : \tau(\mathbf{1}-e_\lambda) \le t\}$.

A non-zero linear subspace $E \subset L_\tau(\mathcal{M})$ with a Banach norm $\|\cdot\|_E$ is called a symmetric (respectively, fully symmetric) space if the conditions $x \in E,\ y \in L_\tau(\mathcal{M}),\ \mu_t(y) \le \mu_t(x)$, for all $t > 0$ (respectively, $\int\limits_0^s \mu_t(y)dt \le \int\limits_0^s \mu_t(x)dt$, for all $s > 0$) imply that $y \in E$ and $\|y\|_E \le \|x\|_E$.

It is known that $L_1(\mathcal{M},\tau) \cap \mathcal{M} \subset E \subset L_1(M,\tau) + \mathcal{M}$ for any symmetric space $E$, where $L_1(\mathcal{M},\tau) = \{x \in L_\tau(\mathcal{M}) : \|x\|_1 = \int\limits_0^\infty \mu_t(x)dt < \infty\}$.

A symmetric space $(E, \|\cdot\|_E)$ is said to have order continuous norm if $\|x_n\|_E \downarrow 0$ for every sequence $\{x_n\}_{n=1}^\infty \subset E$ with $x_n \downarrow 0$.

A linear operator $T : L_1(M,\tau) + \mathcal{M} \to L_1(M,\tau) + \mathcal{M}$ is called a Dunford-Schwartz operator (writing: $T \in DS$), if $\|T(x)\|_1 \le \|x\|_1$ for all $x \in L_1(\mathcal{M},\tau)$ and $\|T(x)\|_{\mathcal{M}} \le \|x\|_{\mathcal{M}}$ for all $x \in \mathcal{M}$. It is known that, $T(E) \subset E$ and $\|T\|_{E\to E} \le 1$ for all $T \in DS$ and for any fully symmetric space $E$.

The following theorem is the criterion for the validity of the mean ergodic theorem in a fully symmetric space $(E, \|\cdot\|_E)$, in the case when $\mathcal{M}$ is an atomic von Neumann algebra.

**Theorem 1.** *Let $\mathcal{M}$ be an arbitrary atomic von Neumann algebra with a faithful normal semifinite trace $\tau$, and let $(E, \|\cdot\|_E) \subset L_\tau(\mathcal{M})$ be a fully symmetric space. Then the following conditions are equivalent:*

*$(i)$. For any operator $T \in DS$ and for any $x \in E$ there exists $\widehat{x} \in E$, such that $\|\frac{1}{n+1}\sum\limits_{k=0}^n T^k(x) - \widehat{x}\|_E \to 0$ as $n \to \infty$;*

*(ii). The space $(E, \|\cdot\|_E)$ has order continuous norm and $E \neq L_1(\mathcal{M}, \tau)$ as sets.*

In the case when an algebra $\mathcal{M}$ is an algebra $\mathcal{B}(\mathcal{H})$ of all bounded linear operators, acting in Hilbert space $H$, the statement of the theorem 1 was obtained in [1].

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## ON ABSOLUTE CONTINUITY OF CONJUGATIONS BETWEEN INTERVAL EXCHANGE MAPS

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In the paper we provide a sufficient condition for absolute continuity of the conjugation between piecewise smooth interval exchange maps. Before formulate our result, define a class of interval exchange maps. Let $\mathcal{A}$ be an alphabet with $d \geq 2$ symbols. Consider the partition of $I$ into $d$ subintervals indexed by $\mathcal{A}$, that is, $\mathcal{P} = \{I_\alpha,\ \alpha \in \mathcal{A}\}$. Let $f : I \to I$ be a bijection. We say that the triple $(f, \mathcal{A}, \mathcal{P})$ is a *generalized interval exchange map* with $d$ intervals (for short g.i.e.m.), if $f|_{I_\alpha}$ is an orientation-preserving homeomorphism for all $\alpha \in \mathcal{A}$.

Let $f : I \to I$ be a g.i.e.m. with alphabet $\mathcal{A}$ and $\pi_0$, $\pi_1 : \mathcal{A} \to \{1, ..., d\}$, be bijections such that $\pi_0(\alpha) < \pi_0(\beta)$, iff $I_\alpha < I_\beta$, and $\pi_1(\alpha) < \pi_1(\beta)$, iff $f(I_\alpha) < f(I_\beta)$. We call pair $\pi = (\pi_0, \pi_1)$ the *combinatorial data* associated to the g.i.e.m. $f$. When appropriate we will also use the notation $\pi = (\pi(1), \pi(2), ..., \pi(d))$ for the combinatorial data of $f$. We always assume that the pair $\pi = (\pi_0, \pi_1)$ is *irreducible*, that is, for all $j \in \{1, ..., d-1\}$ we have: $\pi_0^{-1}(1, ..., j) \neq \pi_1^{-1}(1, ..., j)$. We say that g.i.e.m. $f$ has *cyclic permutation*, if $\pi_0(\{1, 2, ..., d\}) = \{j+1, ..., d, 1, ..., j\}$, for some $1 \leq j \leq d-1$.

We consider generalized interval exchange maps $f$ with *has no connection* codition, that is, $f^m(\partial I_\alpha) \neq \partial I_\beta, \quad \textit{for all} \quad m \geq 1 \ \textit{and} \ \alpha, \beta \in \mathcal{A} \quad \textit{with} \quad \pi_0(\beta) \neq 1$. The latter condition means that the orbits of the left end point of the subintervals $I_\alpha$, $\alpha \in \mathcal{A}$ are disjoint when ever they can be. Irreducible and has no connection generalized interval exchange maps generate a sequence of dynamical partitions $\mathbb{P}_n$ of $I$ (see for instance [1]).

We say that g.i.e.m. $f$ has *k- bounded combinatorics*, if for each $n \in \mathbb{N}$ and $\beta$, $\gamma \in \mathcal{A}$ there exist $n_1, p \geq 0$ with $|n - n_1| < k$ and $|n - n_1 - p| < k$ such that

$$\alpha_{n_1}(\varepsilon_{n_1}) = \beta,\ \alpha_{n_1+p}(1 - \varepsilon_{n_1+p}) = \gamma, \ \ \textit{and}$$

$$\alpha_{n_1+i}(1 - \varepsilon_{n_1+p}) = \alpha_{n_1+i+1}(\varepsilon_{n_1+i}), \quad \textit{for every} \quad 0 \leq i < p.$$

Now we define a class of generalized interval exchange maps. Let $\mathbb{B}^{KO}$ be the set of g.i.e.m. $f : I \to I$ such that

(i) the map $f$ has cyclic permutation;

(ii) the map $f$ has no connection and has $k$- bounded combinatorics.

(iii) on each intervals of continuity of $f$ the map $f$ satisfies Katznelson and Ornstein's smoothness condition: $Df$ is absolutely continuous and $D \ln Df \in L_p$ for some $p > 1$.

Notice that the class $\mathbb{B}^{KO}$ consists of circle homeomorphisms with several break points and with irrational rotation number of bounded type.

Two g.i.e.m. $f_1$, $f_2 \in \mathbb{B}^{KO}$ with the same combinatorics is called break-equivalent, if the following conditions hold true:

(a) the break points of one map $u_i$ are mapped into the break points of the other $v_i$, by a topological conjugacy $h$, satisfying $f_2 = h^{-1} \circ f_1 \circ h$, i.e. $u_i = h(v_i)$;

(b) and the corresponding sizes of breaks, $c_i = \sqrt{f_1'(u_i - 0)/f_1'(u_i + 0)}$ and $\widetilde{c}_i = \sqrt{f_2'(v_i - 0)/f_2'(v_i + 0)}$, are the same, for each $i = 1, ..., k$.

Let $f_1$ and $f_2$ be the break-equivalent g.i.e.m. of class $\mathbb{B}^{KO}$. Consider dynamical partitions $\mathbb{P}_n(f_1)$ and $\mathbb{P}_n(f_2)$ which are generated by the maps $f_1$ and $f_2$. Let $h$ be a conjugation between $f_1$ and $f_2$, that is, $h \circ f_1 = f_2 \circ h$. Denote by $I^{(n)}$ elements of the partition of $\mathbb{P}_n(f_2)$. Since $h$ is a conjugating map between $f_1$ and $f_2$, for any $J^{(n)} \in \mathbb{P}_n(f_1)$ we have $h(J^{(n)}) = I^{(n)}$ and $I^{(n)} \in \mathbb{P}_n(f_2)$. Our main result is the following

**Theorem.** *Let $f_1$ and $f_2$ be the break-equivalent g.i.e.m. of class $\mathbb{B}^{KO}$. Suppose that there exist a sequence $\delta_n$ with $\sum_{n=1}^{\infty} \delta_n^2 < \infty$ such that*

$$\left| \frac{|h(L^{(n)})|}{|h(R^{(n)})|} - \frac{|L^{(n)}|}{|R^{(n)}|} \right| \leq \delta_n,$$

*for each pair of adjacent intervals $L^{(n)}$, $R^{(n)} \in \mathbb{P}_n(f_1)$. Then the conjugating map $h$ is absolutely continuous function.*

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## DERIVATIONS WITH VALUES IN NONCOMMUTATIVE $L_1$-SPACES

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Let $M$ be a $C^*$-algebra and $X$ be a Banach $M$-bimodule. A linear map $\delta : M \to X$ is called a derivation if $\delta(xy) = \delta(x)y + x\delta(y)$ for all $x, y \in M$. Each element $a \in X$ defines a derivation $\delta_a : M \to X$ by $\delta_a(x) = [a, x] = ax - xa,\ x \in M$. Such derivations $\delta_a$ are called *an inner derivations.*

One of the classic problems in operator algebra theory is the following:

**Derivation problem.** Let $M$ be a $C^*$-algebra and $X$ be a Banach $M$-bimodule. For which classes of $M$-bimodules every derivation $\delta : M \to X$ is inner?

It is known the following positive solution of Derivation problem: If $M$ is a reflexive Banach $M$-bimodule then any derivation $\delta : M \to X$ is inner [2].

Note also that if $M = X$ then every derivation $\delta : M \to M$ is continuous. In the case when $M$ ia a von Neumann algebra any derivation $\delta : M \to M$ is inner, that is, $\delta = \delta_a$ for some $a \in M$ [3], in addition, the element $a \in M$ can be chosen such that $\|a\|_M \le \|\delta\|_{M\to M}$. Zsido [4] refined the latter estimate and showed that $\|a\|_M \le \frac{1}{2} \cdot \|\delta\|_{M\to M}$. Below we will establish a variant of the above estimate for derivation from von Neumann algebra $M$ into Banach $M$-bimodule $L_1(M, \tau)$ of all $\tau$-integrable operators affiliated with a finite von Neumann algebra $(M, \tau)$, where $\tau$ is a faithful normal semifinite trace on $M$.

Let $H$ be a complex Hilbert space, let $B(H)$ be the $C^*$algebra of all bounded linear operators on $H$, and let $M \subset B(H)$ be a semifinite von Neumann algebra acting on $H$. A linear operator $x : D(x) \to H$, where domain $D(x)$ of $x$ is a linear subspace of $H$, is said to be *affiliated* with $M$ if $yx \subseteq xy$ for all $y$ from the commutant $M'$ of algebra $M$. Let $\tau$ be a faithful normal semifinite trace on $M$. A densely-defined closed linear operator $x$ affiliated with $\mathcal{M}$ is said to be $\tau$*-measurable* with respect to $\mathcal{M}$ if there exists a sequence projections $\{p_n\}_{n=1}^{\infty} \subset M$ such that $p_n \uparrow \mathbf{1},\ p_n(H) \subset D(x)$ and $\tau(\mathbf{1} - p_n) < \infty$ for every $n = 1, 2, \ldots,$ where $\mathbf{1}$ is the identity operator on $H$. The set $S(M, \tau)$ of all $\tau$-measurable operators is a unital $*$-algebra over the field of complex numbers with respect to strong sum, strong product and involution $x^*$. If $x \in S(M, \tau)$ then $|x| = (x^*x)^{\frac{1}{2}} = \int\limits_0^\infty \lambda\, de_\lambda(|x|) \in S(M, \tau)$, where $e_\lambda(|x|)$ are spectral projections of $|x|$.

An operator $x \in S(M, \tau)$ is called $\tau$-integrable if $\tau(|x|) = \int\limits_0^\infty \lambda\, d\tau(e_\lambda(|x|)) < \infty$. The set $L_1(M, \tau)$ of all $\tau$-integrable operators is a Banach $M$-bimodule with respect to the norm $\|x\|_1 = \tau(|x|)$. In [1] it is established that every derivation $\delta$ from $M$ into any Banach $M$-bimodule $(X(M, \tau), \|\cdot\|_{X(M,\tau)}) \subset S(M, \tau)$ of $\tau$-measurable operators is inner, that is, there is an operator $a \in X(M, \tau)$ such that $\delta = \delta_a$, in addition, $\|a\|_{X(M,\tau)} \le 2\|\delta\|_{M\to X(M,\tau)}$, and if $\delta^* = \delta$ or $\delta^* = -\delta$ then can be chosen an operator $a$ such that $\|a\|_{X(M,\tau)} \le \|\delta\|_{M\to X(M,\tau)}$ (here $\delta^*(x) = (\delta(x^*))^*,\ x \in M$). In particular, every derivation $\delta : M \to L_1(M, \tau)$ is inner.

The following theorem is a variant of Zsido Theorem for derivations $\delta : M \to L_1(M, \tau)$.

**Theorem.** *Let $M$ be a finite von Neumann algebra, and let $\tau$ be a faithful normal* *semifinite trace on $M$. If $\delta^* = \delta : M \to L_1(M,\tau)$ is a derivation then there is an operator $a = a^* \in L_1(M,\tau)$ such that $\delta = \delta_{i \cdot a}$ and $\|a\|_1 \leq 2^{-1}\|\delta\|_{M \to L_1(M,\tau)}$, where $i^2 = -1$.*

Note that in the case of reflexive $M$-bimodules $L_p(M,\tau) = \{x \in S(M,\tau) : \|x\|_p = \big(\tau(|x|^p)\big)^{\frac{1}{p}} < \infty\},\ 1 < p < \infty$, a variant of the above Theorem has not yet been obtained.


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## SOME TOPOLOGICAL PROPERTIES OF SPACE OF THE PERMUTATION DEGREE


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In the work studied are some cardinal and topological properties of the $n$ th permutation degree.

A permutation group $X$ is the group of all permutations (i.e. one-one and onto mappings $X \to X$ ). A permutation group of a set $X$ is usually denoted by $S(X)$ . If $X = \{1, 2, ...., n\}$, $S(X)$ is denoted by $S_n$ , as well.

Let $X^n$ be the $n$ th power of a compact $X$. The permutation group $S_n$ of all permutations, acts on the $n$ th power $X^n$ as permutation of coordinates. The set of all orbits of this action with quotient topology we denote by $SP^nX$.

Consider as a factor mapping $\pi_n^s : X^n \to SP^nX$ put the point $x = (x_1, x_2, ..., x_n) \in X^n$ to the point orbit of this point.

Thus, points of the space $SP^nX$ are finite subsets (equivalence classes) of the product $X^n$. Thus two points $(x_1, x_2, ..., x_n), (y_1, y_2, ..., y_n) \in X^n$ , are considered to be equivalent if there is a permutation $\sigma \in S_n$ such that $y_i = x_{\sigma(i)}$ for all $i = 1, 2, ..., n$. The space $SP^nX$ is called the $n$ -permutation degree of a spaces $X$ . Equivalence relations by which we obtained spaces $SP^nX$ and $exp_nX$ , is called the symmetric and hypersymmetric equivalence relations, respectively.

The concept of a permutation degree has generalizations. Let $G$ be any subgroup of the group $S_n$ . Then it also acts on $X^n$ as group of permutations of coordinates. Consequently, it generates a $G-$ symmetric equivalence relation on $X^n$. The quotient space of the product $X^n$

under the $G-$symmetric equivalence relation, is called $G-$ permutation degree of the space $X$ and is denoted by $SP^n_G X$. An operation $SP^n_G$ is also the covariant functor in the category of compacts and is said to be a functor of $G-$permutation degree. If $G = S_n$ then $SP^n_G = SP^n$. If the group $G$ consists only of unique element then $SP^n_G X = X^n$.

Moreover, if $G_1 \subset G_2$ for subgroups $G_1, G_2$ of the permutation group $S_n$ then we get a sequence of the factorization of functors:

$$X^n \to SP^n_{G_1} X \to SP^n_{G_2} X \to SP^n X \to exp_n X.$$

**Theorem 1.** *Let $SP^n_G X$ be a topological space with the quotient topology and $\pi^s_n : X^n \to SP^n X$ as above. The following three statements hold:*

*(1) $\pi^s_n$ is onto;*

*(2) $\pi^s_n$ is open;*

*(3) $\pi^s_n$ is closed.*

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## A WEIGHT OPTIMAL QUADRATURE FORMULA IN $L_2^{(2)}(0, \pi)$ SPACE

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It is known that integrals of the type

$$I(f) = \int_{\mathbb{S}^2} f(\xi) dS^2(\xi)$$

occur in a wide variety of physical applications, where $dS^2(\eta)$ is an element of $\mathbb{S}^2$, $\xi = (x, y, z) \in \mathbb{S}^2$. There are many of approaches to developing numerical integration formulas for the integral over the sphere. We suggest the methods based on $I(f)$ being represented as a double integral using spherical coordinates, followed by application of optimal quadrature formulas. Numerical integration of $I(f)$ over two dimensional unit sphere $\mathbb{S}^2$ is reduced to calculation of the following integral

$$\int_0^\pi \varphi(x) \sin x dx,$$

where $\varphi$ is a continuous function.[1]

The present work is devoted to construction of optimal quadrature formulas of the form

$$\int_0^\pi \varphi(x) \sin x dx \cong \sum_{\beta=0}^{N} C[\beta] \varphi(h\beta), \tag{1}$$

here $h = \pi/N$, $N$ is a natural number and $C[\beta]$ are coefficients, $\varphi$ is an element of the space $L_2^{(2)}(0,\pi)$ which is the Sobolev space of square integrable functions with the second generalized derivative.

The error

$$(\ell,\varphi) = \int_0^\pi \sin x\varphi(x)dx - \sum_{\beta=0}^{N} C[\beta]\varphi(h\beta)$$

of the formula (1) is estimated by the norm $\left\|\ell|L_2^{(2)*}\right\|$ of the error functional $\ell$ in the conjugate space $L_2^{(2)*}(0,\pi)$.

Furthermore, the norm of the error functional $\ell$ depends on the coefficients $C[\beta]$. Finding the coefficients when the nodes are fixed is linear problem. Therefore we find the following quantity

$$\left\|\mathring{\ell}|L_2^{(2)*}\right\| = \inf_{C[\beta]} \left\|\ell|L_2^{(2)*}\right\|$$

If $\left\|\mathring{\ell}|L_2^{(2)*}\right\|$ is found then the functional $\mathring{\ell}$ is said to be correspond to the optimal quadrature formula (1) in $L_2^{(2)}(0,\pi)$ and the corresponding coefficients are called optimal.

Here we find analytic formulas for optimal coefficients $C[\beta]$ of the formula (1) which are very useful for application.

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## INFLUENCE FACTORS ON ENERGY CONVERSION IN LOW POWER SUPPLIES: TECHNOLOGY + MATHEMATICS + MEDICINE


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Green technologies involve the use of radioisotope waste from nuclear power plants to generate electricity by directly converting the energy of radiochemical transformations through semiconductor structures. The generation of multi-nanolevel electricity by means of the heterostructure of silicon carbide on the silicon substrate was tested (on the example of a radioisotope medical product containing C-14).

Authors discuss results and optimization of the power supply for MEMS/NEMS devices, based on SiC/Si structures, which are tested to be used as the beta-decay energy converters of radioactive C-14 into electrical energy. This is based on the silicon carbide obtaining by the self-organizing mono 3C-SiC endotaxy on the Si substrate. The idea is the C-14 atoms including in molecules in the silicon carbide structure by this technology, which will increase the efficiency of the converter due to the greater intensity of electron-hole pairs generation rate in the space charge region and the mobility of the radionuclide.

The content of C-14 in the SiC phase was determined at the doping level, but this was sufficient to generate nonequilibrium carriers in the p-n region of the transition in silicon carbide and their separation by the space charge region field, while dark currents from 16 to 90 nA were observed in the external circuit. The measurements were carried out on the stand excluding electromagnetic interference without the formation of metallization of the contact pads by the tip probe method. Idling voltage was amounted to 1.6 mV. The thickness of the activated n-SiC film in the heterostructure is 5 $\mu$m.

It is known that irradiation of n-6H-SiC by electrons with energies of 2-3.5-4 MeV formes defective centers with activation energy 0.6-1.1 eV near the conduction band. There are centers also in politype modification of 4-H-SiC crystal lattice with another quantity and energies of activation. And centers are annealed. Such a problem may occur if the 3C-SiC/Si structures are alloying with C-14, generated by the endotaxi technology through the solid-phase transformation reactions of por-Si-phase in por-SiC-phase. In this case, the annealing can be produced at lower temperatures, ensuring operability based on radiation safety and of sensor resistance change.

## ISOMETRIES OF BANACH-KANTOROVICH $L_p$-SPACES

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The development of the theory of integration for measures with values in order-complete vector lattices $F$ made it possible to construct new informative examples of Banach-Kantorovich spaces, i.e. the linear spaces with complete norm, taking values in $F$. The important examples of such Banach-Kantorovich spaces are $L_p$-spaces $L_p(\nabla, m)$, $1 \leq p < \infty$, constructed by vector-valued Maharam measure $m$, which defined on complete Boolean algebra $\nabla$ [2, Chapter 3, Section 3.6.4]. In the case when $F$ is the field of real numbers $\mathbb{R}$, these spaces $L_p(\nabla, m)$ are the classical $L_p$-spaces $L_p(\Omega, \mathcal{A}, \mu)$ of all classes $[f]$ of equal almost everywhere measurable functions $f$, defined on a measure space $(\Omega, \mathcal{A}, \mu)$ with finite measure $\mu$, such that $\|f\|_p = \big(\int\limits_{\Omega} |f|^p \, d\mu\big)^{\frac{1}{p}} < \infty$.

One of the important results in the general theory of isometries of Banach spaces is the following description of all surjective linear isometries $U : L_p(\Omega, \mathcal{A}, \mu) \to L_p(\Omega, \mathcal{A}, \mu)$ in the case when $p \neq 2$ (see, e.g. [1, Chapter 3, Theorem 3.2.5]): There exist an isomorphism $\Phi$ of algebra $L_0(\Omega, \mathcal{A}, \mu)$ and a function $h \in L_p(\Omega, \mathcal{A}, \mu)$ such that $U(f) = h \cdot \Phi(f)$ for all $f \in L_p(\Omega, \mathcal{A}, \mu)$ (here $L_0(\Omega, \mathcal{A}, \mu)$ is an algebra of all classes $[f]$ of equal almost everywhere measurable functions $f$, defined on a measure space $(\Omega, \mathcal{A}, \mu)$). Below we will establish a version of the above statement for surjective linear isometries of Banach-Kantorovich spaces $L_p(\nabla, m)$, $p \neq 2$.

Let $\nabla$ be an arbitrary Boolean algebra, let $Q(\nabla)$ be the Stone compact of $\nabla$, and let $C_\infty(Q(\nabla))$ be the commutative unital algebra over the field $\mathbb{R}$ of all continuous functions $x : Q(\nabla) \to [-\infty, +\infty]$, assuming the values $\pm\infty$ possibly on a nowhere-dense subset of $Q(\nabla)$ (see e.g. [2, Chapter 1, Section 1.4.2]). It is well-known that $C_\infty(Q(\nabla))$ is complete vector lattice with respect to the partial order $c \leq d \Longleftrightarrow c(t) \leq d(t)$ for all $t \in \{s \in Q(\nabla) : \ |c(s)| < +\infty, \ |d(s)| < +\infty\}$. Let $m : \nabla \to L_0(\Omega, \mathcal{A}, \mu)$ be strictly positive $\sigma$-additive measure with Maharam property: for all $e \in \nabla$, $0 \leq \lambda \leq m(e)$, $\lambda \in L_0(\Omega, \mathcal{A}, \mu)$ there exists $g \in \nabla$, $g \leq e$, such that $m(g) = \lambda$. For any step element $f = \sum\limits_{i=1}^{n} c_i e_i$, $c_i \in \mathbb{R}$, $e_i \in \nabla$, $e_i e_j = 0$, $i \neq j$, $i, j = 1, \ldots, n$, we set $\int\limits_{\nabla} f \ dm = \sum\limits_{i=1}^{n} c_i m(e_i) \in L_0(\Omega, \mathcal{A}, \mu)$. Let $0 \leq f \in C_\infty(Q(\nabla))$ and there exists a sequence of step elements $0 \leq f_n \uparrow f$ such that $\int\limits_{\nabla} f_n \ dm \leq K \in L_0(\Omega, \mathcal{A}, \mu)$ for all $n$. In this case an element $f$ is called integrable with respect to measure $m$, and we put $\int\limits_{\nabla} f \ dm = \sup\limits_{n \geq 1} \int\limits_{\nabla} f_n \ dm$. If $f$ is an arbitrary element in $C_\infty(Q(\nabla))$ and $f_+ = f \vee 0$, $f_- = (-f) \vee 0$ its positive and negative parts, respectively, then $f$ is called integrable with respect to measure $m$, if $f_+$ and $f_-$ are integrable with respect to measure $m$. In this case we put $\int\limits_{\nabla} f \ dm = \int\limits_{\nabla} f_+ \ dm - \int\limits_{\nabla} f_- \ dm \in L_0(\Omega, \mathcal{A}, \mu)$.

For every $1 \leq p < \infty$ we put $L_p(\nabla, m) = \{f \in C_\infty(Q(\nabla)) : \|f\|_p = \big(\int\limits_{\nabla} |f|^p \ d\mu\big)^{\frac{1}{p}} \in L_0(\Omega, \mathcal{A}, \mu)\}$. It is known that $(L_p(\nabla, m), \| \cdot \|_p)$ is a Banach-Kantorovich space (see e.g. [2, Chapter 3, Section 3.6.4]).

The following theorem gives a complete description of all surjective linear isometries $U : L_p(\nabla, m) \to L_p(\nabla, m)$ in the case when $p \neq 2$.

**Theorem.** *A surjective linear map $U : L_p(\nabla, m) \to L_p(\nabla, m)$, $1 \leq p < \infty$, $p \neq 2$, is isometry if and only if there exists an isomorphism $\Phi$ of algebra $C_\infty(Q(\nabla))$ and a function $h \in L_p(\nabla, m)$ such that $U(f) = h \cdot \Phi(f)$ for all $f \in L_p(\nabla, m)$, where $h$ satisfies $|h|^p = \frac{d(m \circ \Phi^{-1})}{dm}$, where $\frac{d(m \circ \Phi^{-1})}{dm}$ is the Radon-Nikodym derivation of the measure $(m \circ \Phi^{-1})(e) = m(\Phi^{-1}(e))$, $e \in \nabla$, with respect to the measure $m$.*


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## NORMALITY OF COMPLETELY ADDITIVE LINEAR MAPPINGS OF IDEAL SUBSPACES IN THE ALGEBRA $C_\infty(Q)$


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It is well known that any positive completely additive linear mapping acting in the algebra $\mathcal{L}_\infty(\Omega, \mathcal{A}, \mu)$ is a normal mapping. In this note we establish the normality of each completely additive positive linear mapping acting in ideal subspaces of the algebra $C_\infty(Q)$.

Let $\nabla$ be an arbitrary Boolean algebra, let $Q(\nabla)$ be the Stone compact of $\nabla$, and let $C_\infty(Q(\nabla))$ be the commutative unital algebra over the field real numbers $\mathbb{R}$ of all continuous functions $x : Q(\nabla) \to [-\infty, +\infty]$, assuming the values $\pm\infty$ possibly on a nowhere-dense subset of $Q(\nabla)$ (see e.g. [1, Section 1.4.2]). It is well-known that $C_\infty(Q(\nabla))$ is complete vector lattice with respect to the partial order $c \leq d \Longleftrightarrow c(t) \leq d(t)$ for all $t \in \{s \in Q(\nabla) : |c(s)| < +\infty,\ |d(s)| < +\infty\}$.

A non-zero linear subspace $E \subset C_\infty(Q(\nabla))$ is called an ideal space if the conditions $x \in E,\ y \in C_\infty(Q(\nabla)),\ |y| \leq |x|$, imply that $y \in E$.

Let $E \supset \nabla$ be an ideal subspace in $C_\infty(Q(\nabla))$. Positive linear mapping $T : E \to E$ is called completely additive (respectively, normal) if $T(e_\alpha) \uparrow T(e)$ for any increasing net $\{e_\alpha\} \subset \nabla$ of idempotents with $e_\alpha \uparrow e$ (respectively, $T(x_\alpha) \uparrow T(x)$ for any increasing net $\{x_\alpha\} \subset E$ with $x_\alpha \uparrow x$). It is clear that any normal mapping is completely additive mapping.

**Theorem 1.** *Any completely additive mapping* $T : E \to E$ *is normal mapping.*

**Corollary 1.** *Any completely additive homomorphism* $\Phi : C_\infty(Q(\nabla)) \to C_\infty(Q(\nabla))$ *is normal mapping.*

Let $(\Omega, \mathcal{A}, \mu)$ be a measurable space with a complete $\sigma$-finite measure, let $\nabla$ be the complete Boolean algebra of all equivalence classes $[A]$ of $A \in \mathcal{A}$, and let $L^0(\nabla)$ be a linear space of equivalence classes of all real valued measurable functions on $(\Omega, \mathcal{A}, \mu)$. It is clear that algebras $L^0(\nabla)$ and $C_\infty(Q(\nabla))$ are isomorphic. Let $t_\mu$ be the locally measure topology in $L^0(\nabla)$. Convergence of the net $\{x_\alpha\}_{\alpha\in B} \subset L^0(\nabla)$ to $x \in L^0(\nabla)$ with respect to the topology $t_\mu$ means that $x_\alpha \cdot \chi_A \longrightarrow x \cdot \chi_A$ with respect to the measure $\mu$ for any set $A \in \mathcal{A}$ with $\mu(A) < \infty$, where $\chi_A$ is the characteristic function of the set $A \in \mathcal{A}$.

**Corollary 2.** *Any completely additive* $*$*-homomorphism* $\Phi : L^0(\nabla) \to L^0(\nabla)$ *is continuous with respect to the locally measure topology* $t_\mu$*, in particular, the graph* $\Gamma_\Phi = \{(x, \Phi(x)) : x \in L^0(\nabla)\}$ *is closed in* $(L^0(\nabla), t_\mu) \times (L^0(\nabla), t_\mu)$.

The following Theorem is converse to Corollary 2.

**Theorem 2.** *Let* $\Phi : L^0(\nabla) \to L^0(\nabla)$ *be a homomorphism, and let the graph* $\Gamma_\Phi$ *be closed in* $(L^0(\nabla), t_\mu) \times (L^0(\nabla), t_\mu)$*. Then* $\Phi$ *is normal homomorphism and* $\Phi$ *is continuous with respect to the topology* $t_\mu$.

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## MARCINKEVICH-KANTOROVICH SPACES


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The development of the theory of integration for measure with values in the Dedekind complete Riesz space has inspired the study new classes of Banach-Kantorovich spaces that are "vector" variants of the classical $Lp$-spaces [2, Chapter 3, §3.5] and Orlicz spaces [1]. Below we will given construct of Marcinkevich-Kantorovich spaces associated with the Maharam measure that takes values in the Dedekind complete Riesz space $L^0(\Omega)$ of all equivalence classes of real measurable functions on $(\Omega, \mathcal{A}, \mu)$, where $(\Omega, \mathcal{A}, \mu)$ is measure space with a complete $\sigma$-finite measure $\mu$.

Let $\nabla$ be an arbitrary complete Boolean algebra, let $Q(\nabla)$ be a Stone compact corresponding to $\nabla$, and let $C_\infty(Q(\nabla))$ be an algebra of all continuous functions $x : Q(\nabla) \to [-\infty, +\infty]$, taking the values $\pm\infty$ only on nowhere dense sets.

Consider a strongly positive completely additive $L^0(\Omega)$-valued measure $m$ defined on Boolean algebra $\nabla$. The measure $m$ is called a Maharam measure if for any $e \in \nabla$, $0 \le \lambda \le m(e)$, $\lambda \in L^0(\Omega)$, there exists $g \in \nabla$, $g \le e$, such that $m(g) = \lambda$.

For any $x \in C_\infty(Q(\nabla))$ and any $t \in (0, \infty)$ set $\eta_x(t) = m(\{|x| > t\})$, where $\{|x| > t\} = \{s \in Q(\nabla) : |x(s)| > t\}$. It is clear that $0 \le \eta_x(t) \in L^0(\Omega)$ for all $t \in (0, \infty)$. Consider the product $((0, +\infty), \Sigma, \nu) \otimes (\Omega, \mathcal{A}, \mu)$ of the measure spaces $((0, +\infty), \Sigma, \nu)$ and $(\Omega, \mathcal{A}, \mu)$, where $\Sigma$ is the $\sigma$-algebra of Lebesgue measurable sets from $(0, +\infty)$ and $\nu$ is a Lebesgue measure on $\Sigma$. A non-negative measurable function $m_x(t, \omega) : ((0, +\infty), \Sigma, \nu) \otimes (\Omega, \mathcal{A}, \mu) \to (0, +\infty)$ is called non-increasing $m$-rearrangement of the element $x \in C_\infty(Q(\nabla))$, if $m_x(t, \omega) = \inf\{\tau > 0 : \eta_x(\tau)(\omega) \le t\}$, $\omega \in \Omega$, $t > 0$.

Let $\psi$ be a concave function on $[0, \infty)$, $\psi(0) = 0$ and $\psi(t) > 0$ for all $t > 0$, and let

$$M_\psi(0, +\infty) = \left\{ f \in L^0((0, +\infty), \Sigma, \nu)) : \ \|f\|_{M_\psi} = \sup_{0<s<\infty} \frac{1}{\psi(s)} \int_0^s f^*(t)dt < \infty \right\}$$

be the classical function Marcinkevich space (see e.g. [3, Chapter II, §5]), where $f^*(t) = \inf\{\tau > 0 : \nu(\{|f| > \tau\}) \le t\}$ is a non-increasing $\nu$-rearrangement of the function $f$ from $L^0((0, +\infty), \Sigma, \nu))$. Set

$$M_\psi(\nabla, \mu) = \{x \in C_\infty(Q(\nabla)) : m_x(t, \omega) \in M_\psi(0, +\infty) \text{ for } \mu\text{-almost everywhere } \omega \in \Omega\},$$

and let $\|x\|_{M_\psi}(\omega) = \|m_x(t, \omega)\|_{M_\psi}$, $x \in M_\psi(\nabla, \mu)$.

The following theorem is a vector-valued version of the well-known theorem for classical function spaces Marcinkevich (see, e.g. [3, Chapter II, §5]).

**Theorem.** *$M_\psi(\nabla, \mu)$ is $L^0(\Omega)$-module and a regular vector sublattice in $C_\infty(Q(\nabla))$, and a mapping $\|\cdot\|_{M_\psi} : M_\psi(\nabla, \mu) \to L^0(\Omega)$ is $L^0(\Omega)$-valued a Banach norm on $M_\psi(\nabla, \mu)$.*

Thus the pair $(M_\psi(\nabla, \mu), \|\cdot\|_{M_\psi})$ is new example of Banach-Kantorovich lattices which is naturally called Marcinkevich-Kantorovich space.

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## EIGENVALUES OF A FAMILY OF $2 \times 2$ OPERATOR MATRICES

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Block operator matrices are matrices where the entries are linear operators between Banach or Hilbert spaces [1]. One special class of block operator matrices are Hamiltonians associated with systems of non-conserved number of quasi-particles on a lattice. They arise, for example, in the theory of solid-state physics, quantum field theory and statistical physics.

Let $\mathcal{H}_0 := \mathbb{C}$ be the field of complex numbers (zero-particle subspace of a Fock space) and $\mathcal{H}_1 := L_2(\mathbb{T}^3)$ be the Hilbert space of square-integrable (complex) functions defined on be the three-dimensional torus $\mathbb{T}^3$ (one-particle subspace of a Fock space).

In the Hilbert space $\mathcal{H} := \mathcal{H}_0 \oplus \mathcal{H}_1$ we consider the following family of $2 \times 2$ operator matrices

$$\mathcal{A}_\mu(k) := \begin{pmatrix} A_{00}(k) & \mu A_{01} \\ \mu A_{01}^* & A_{11}(k) \end{pmatrix},$$

where $A_{ii}(k) : \mathcal{H}_i \to \mathcal{H}_i$, $i = 0, 1$, $k \in \mathbb{T}^3$ and $A_{01} : \mathcal{H}_1 \to \mathcal{H}_0$ are defined by the rules

$$A_{00}(k) f_0 = w_0(k) f_0, \quad A_{01} f_1 = \int_{\mathbb{T}^3} f_1(t) dt, \quad (A_{11}(k) f_1)(p) = w_1(k, p) f_1(p).$$

Here $f_i \in \mathcal{H}_i$, $i = 0, 1$, $\mu > 0$ is a coupling constant, the functions $w_0(\cdot)$ and $w_1(\cdot, \cdot)$ have the form

$$w_0(k) := \varepsilon(k) + \gamma, \quad w_1(k, p) := \varepsilon(k) + \varepsilon(\frac{1}{2}(k + p)) + \varepsilon(p)$$

with $\gamma \in \mathbb{R}$ and the dispersion function $\varepsilon(\cdot)$ is defined by

$$\varepsilon(k) := \sum_{i=1}^{3} (1 - \cos k_i), \; k = (k_1, k_2, k_3) \in \mathbb{T}^3,$$

$A_{01}^*$ denotes the adjoint operator to $A_{01}$. Under these assumptions the operator matrix $\mathcal{A}_\mu(k)$ is a bounded and self-adjoint in $\mathcal{H}$.

Set $\overline{0} := (0, 0, 0)$, $\overline{\pi} := (\pi, \pi, \pi)$. Note that $\sigma_{\text{ess}}(\mathcal{A}_\mu(\overline{\pi})) = [8\frac{5}{8}; 18]$.

Denote

$$\mu_0(\gamma) := \sqrt{12 - \gamma} \left( \int_{\mathbb{T}^3} \frac{dt}{w_1(\overline{0}, t)} \right)^{-1/2} \text{ for } \gamma < 12.$$

The main result of this note is the following

**Theorem 1.** A) *If* $\gamma \geq 12$, *then for any* $\mu > 0$ *the operator* $\mathcal{A}_\mu(\overline{\pi})$ *has no eigenvalues, bigger than* 18.

B) *Let* $\gamma < 12$.
B$_1$) *For any* $\mu \in (0;\mu_0(\gamma)]$, *the operator* $\mathcal{A}_\mu(\overline{\pi})$ *has no eigenvalues bigger than* 18;
B$_2$) *For any* $\mu > \mu_0(\gamma)$ *the operator* $\mathcal{A}_\mu(\overline{\pi})$ *has a unique eigenvalue in* $(18;+\infty)$.


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## MATHEMATICS OF ION AND PLASMA MULTIPHASE FLOW IN THE PLASMA ELECTRIC GENERATOR + TECHNOLOGY


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To solve the problem of controlled synthesis of light nuclei, a technique and technology for the creation and formation of electronically controlled ion flows in a magnetic field by grouping the flows by discretization and specifying certain laws of succession for ion flows [1,2] is developed. To fall into the range of nuclear forces in cases where charged particles or ions are used, their kinetic energy should be sufficient to overcome the Coulomb repulsion of the nucleus. In cases where the energy of a charged particle or ion is less than the height of the Coulomb barrier, the probability of a nuclear reaction will tend to zero [2]. The particle beams of the required energies are easily obtained in modern accelerators. If the energy of the particle is insufficient to overcome the Coulomb barrier, it will experience an elastic scattering in the Coulomb field of the nucleus, described by the Rutherford formula. Light nuclei where the Coulomb barrier is low, this rule does not apply. To generate a stable nuclear fusion reaction, a number of conditions must be met:
1) the total energy of the incoming particle and the target must be higher than the energy of the Coulomb repulsion
2) the density of $n_i$ of the incoming flow and the density of the target $n_m$ for the pulse mode should be equal to or higher than $n_i = n_m \geq 10^{22}\text{cm}^{-3}$.
3) retention time in the magneto-optical chamber $\tau \geq 1$ s.
4) ion energy in the incoming flow $W_i \geq 200$ KeV.

The generation of the dense flow of protons, deuterium or tritium atoms for the neutrons synthesis on the ion-plasma target of deuterium, tritium or lithium occurs as the result of primary flow compaction and discretization by software-defined concentration and average energy of the flow. Flows are formed in strictly specified parameters: $T$ – sequence period, $n$ – concentration and the frequency of discrete flows $\omega$.

# THE LIMIT THEOREM FOR HITTING TIMES OF CIRCLE MAPS WITH SINGULARITIES

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In this paper we study distribution functions for normalized hitting times for circle homeomorphisms with single break point. Consider orientation preserving circle homeomorphism $f$ with irrational rotation number $\rho$. The continued fraction expansion $\rho$ is $\rho = [1, 1, \dots, 1, \dots] = \frac{\sqrt{5}-1}{2}$. Set $\frac{p_n}{q_n} = [1, 1, \dots, 1]$, $n \geq 1$. The numbers $q_n$, $n \geq 1$ satisfies difference equation $q_{n+1} = q_n + q_{n-1}$, $q_0 = 1$, $q_1 = 1$. Take an arbitrary point $x_0 \in S^1 = \mathbb{R}^1/\mathbb{Z}^1 \simeq [0, 1)$. We denote $x_i = f^i(x_0)$, $i \geq 1$. Let $\Delta_0^{(n)}(x_0)$ be a closed segment connecting the points $x_0$ and $x_{q_n}$. Set $\Delta_i^{(n)}(x_0) = f^i(\Delta_0^{(n)}(x_0))$, $i \geq 1$. Fix $c \in (0, 1)$. For each $n \geq 1$ we define the sequence $c_n$ by $c_n = cx_{q_n}$. We denote by $I_{n,c}$ interval $[0, c_n)$. Define the first return time function $R_{n,c} : I_{n,c} \to \mathbb{N}$:

$$R_{n,c}(x) = \min\{j \geq 1 : T^j x \in I_{n,c}\}.$$

Now we define the hitting time $E_{n,c} : S^1 \to \mathbb{N}$:

$$E_{n,c}(x) = \min\{j \geq 1 : T^j x \in I_{n,c}\}, \;\; x \in S^1.$$

Using the structure of dynamical partitions it is easy to see that $E_{n,c}(x)$ takes values from 1 to $q_{n+3}$ [1]. Next we introduce the normalized hitting times $\overline{E}_{n,c}(x)$ by $\overline{E}_{n,c}(x) = \frac{1}{q_{n+3}} E_{n,c}(x)$. Obviously, that function $\overline{E}_{n,c}(x)$ will be a random variable taking values in $[0, 1]$. We denote by $\Phi_{n,c}(t)$ distribution function of $\overline{E}_{n,c}(x)$.

De Faria and Z. Coelo [2,3] proved that the limit distribution of a converging subsequence $\{F_n^{(1)}(t), \; n = 1, 2, ...\}$, depending on the rotation number $\rho$, is either a uniform distribution, or a continuous piecewise linear distribution on $[0, 1]$. For $k > 1$ limit of sequence $\{F_{n_s}^{(k)}(t), \; s = 1, 2, ...\}$ is or distribution for random variable $\xi \equiv 1$, or step distribution with two break points. Now we formulate the next theorem.

**Theorem 1.** *Let $f$ be a circle homeomorphism with a single break point $x_0 = 0$, $\{\Phi_{n,c}(t)\}_{n=1}^{\infty}$ – the sequence of distribution functions with respect to the Lebesgue measure on the circle, corresponding to the first hitting time in $I_{n,c}$. Then*

*1) for all $t \in R^1$ there exist limit*

$$\lim_{n\to\infty} \Phi_{n,c}(t) = \Phi_c(t),$$

*where $\Phi_c(t) = 0$, if $t \leq 0$, and $\Phi_c(t) = 1$, if $t > 1$ ;*

*2) The limit distribution function $\Phi_c(t)$ is a strictly increasing on $[0, 1]$ and continuous on $R^1$ .*

*3) $\Phi_c(t)$ is singular function on $[0, 1]$, i.e. $\Phi_c'(t) = 0$ a.e. on $[0, 1]$ w.r.t. Lebesgue measure.*

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## QUALITATIVE ANALYSIS OF MATHEMATICAL MODELS BASED ON Z-NUMBER

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Construct a $Z$-solution to the prediction problem.

Let the notion of (forecast accuracy) correspond to a fuzzy set with the membership function of the form

$$\mu_A(y) = \lambda\left(\{y_i\}, \{x_i\}, y\right),\ i = 1, .., n,$$

where $\{y_i\}, \{x_i\}$ is given points, $y(x)$ is desired function. Similarly, the membership function of the notion (forecast quality of a mathematical model) is constructed $\mu_B(y)$.

As usual, a fuzzy solution is some combination of source primary information.

$$\mu(y) = \mu_A^*(y) * \mu_B^*(y).$$

The type of membership functions and the way they are associated depends on a priori information.

The goal is to predict a known sequence. $\{x_i\},\ i = 1, .., n$, i.e. create additional sequence $\{x_i\}$, i=n+1, ..., n+m. Need to find a discrete function $y(i)$, which continues this series of values.

The simplest a priori information will be the knowledge that the type of function and its quality can be determined at different intervals-learning interval and exam interval.

Thus, the concept of (type of function) (model accuracy) corresponds to a fuzzy set with the membership function

$$\mu_A(y) = \exp\left(-\sum_{i=1}^{k}\left(x_i - \inf_y y(i)\right)^2\right).$$

The concept of (quality prediction) meets a fuzzy set with the function of belonging

$$\mu_B(y) = \exp\left(-\sum_{i=k+1}^{n}\left(x_i - y(i)\right)^2\right).$$

Fuzzy solution is a combination of source primary information.

$$\mu(y) = \sqrt{k_1 k_2^{\eta} \mu_A(y) \mu_B^{\eta}(y)},$$

where $\eta$ - parameter of confidence in the quality of the forecast.

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## ON SOLUTIONS OF HAMMERSTEIN INTEGRAL EQUATIONS WITH DEGENERATE KERNEL


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We consider Hammerstein's operator:

$$f(x) = \lambda \int_a^b K(x, y) f^2(y) dy + g(x), \tag{1}$$

where $K(x, y) = \sum_{k=1}^n a_k(x) b_k(y) \in C[a, b]^2$; $f(x), g(x) \in C[a, b]$ and $\lambda \in \mathbb{R}$.

Then, equation (1) can be written as:

$$f(x) = \lambda \sum_{k=1}^{n} a_k(x) c_k + g(x), \tag{2}$$

where $c_k = \int_a^b b_k(y) f^2(y) dy$, $k = \overline{1, n}$.

Consequently, from the (1) and (2) we get systems of quadratic algebraic equations corresponding to $c_1, c_2, ..., c_n$:

$$c_k = \int_a^b b_k(y) \left( \lambda \sum_{i=1}^{n} c_i a_i(y) + g(y) \right)^2 dy, \ k = \overline{1, n} \tag{3}$$

and (3) is a quadratic operator on $\mathbb{R}^n$.

Let $B(\cdot, \cdot) : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n$ be a symmetric bilinear operator. The quadratic operator $Q : \mathbb{R}^n \to \mathbb{R}^n$ is defined by the equation: $Q(x) = B(x, x)$.

**Definition.** A quadratic operator $Q$ is called elliptic if there exists a linear continuous functional $f$such that $f(Q(x))$ is a positive definite quadratic form.

**Theorem.** *If (3) is elliptic type, then:*

1. *The set of solutions of (3) is bounded;*
2. *Let $X$ be the set of all solutions of (3). Then $X \subset \mathrm{extr} M$, where $M \subset C[a, b]$ is a convex bounded set.*

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## ON ASYMPTOTICAL BEHAVIOR OF TRAJECTORIES OF VOLTERRA TYPE OPERATORS

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It is known, a Volterra quadratic stochastic operator on a simplex $S^{m-1}$ can always be represented as

$$x'_k = x_k \left(1 + \sum_{i=1}^{m} a_{ki} x_i\right), \quad k = \overline{1,m}, \tag{1}$$

where $a_{ki} = -a_{ik}$, $|a_{ki}| \leq 1$.

Let $V : S^{m-1} \to S^{m-1}$ be a Volterra type operator, and $x^0 \in S^{m-1}$. A sequence $\{x^{(n)}\}$, where $x^{(n)} = V^n x^0$, is called the trajectory for $n \in \mathbb{Z}$, the positive (negative) trajectory for $n \in \mathbb{N}$ $(-n \in \mathbb{N})$.

Let $I = \{1, 2, ..., m\}$, $\delta_{ij}$ be the Kronecker symbol $(i, j \in I)$ and $\alpha \in I$ be arbitrary nonempty set. Denote by $e_i = (\delta_{i1}, \ldots, \delta_{im})$ the vertices of the simplex $S^{m-1}$. The face $\Gamma_\alpha$ is defined as the convex hull of a set of vertices $\{e_i\}_{i\in\alpha}$.

**Definition 1.** An ordered pair of vertices is called the edge of the digraph. A digraph is called a tournament if for any two different vertices $i$ and $k$, one and only one of the ordered pairs $(i, k)$ or $(k, i)$ is an edge of the digraph.

**Definition 2.** A tournament is called strong if one can get from any vertex to any other one, taking into account the direction of the ribs .

The transitivity of a tournament means that any sub-tournament of this tournament is not strong.

**Theorem.** *1) If a tournament is strong, then there exists a trajectory which does not converge.*

*2) If a tournament is transitive, then any trajectory converges to one of the simplex vertices.*

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# CONTROL OF THE HEAT CHANGING PROCESS WITH A GIVEN FLOW AT THE BOUNDARY

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In this paper, we investigates the problem of boundary control of the heat exchange process by a given heat flux on a part of the boundary of the region.

Let function $u(x,t)$ satisfies to the equation

$$u_t = u_{xx} \tag{4}$$

in the domain $\Omega = \{(x,t) : 0 < t < T, -\pi < x < \pi\}$ and satisfies to the initial condition

$$u|_t = 0, \quad x \in [= \pi, \pi]$$

and boundary conditions

$$u_x|_{x=-\pi} = \mu(t), \quad u_x|_{x=\pi} = 0, \quad t \in [0,T]$$

**Definition 1.** By the solution of the problem (1)-(3), we understand the continuous function $u(x,t)$ on $\bar{\Omega}$, with continuous derivative with respect to $x$ on $[-\pi,\pi]$, and also having continuous derivatives included in equation (1) on the $\Omega$ and satisfying equation (1) and conditions (2), (3).

**Definition 2.** A control $\mu(t)$ is called admissible if the function $\mu(t)$ satisfies the condition

$$|\mu(t)| \leq 1.$$

**Problem.** For a given mean value of the solution $\theta(t)$, find a control $\mu(t)$ that ensures the given equality:

$$\int_{-\pi}^{0} u(x,t)dx = \theta(t), \quad t > 0.$$

From the papers dedicated on boundary controls for various equations, one can specify works of V.A. Ilin and E.I. Moiseev, S. Albeverio, Sh. Alimov, H.O. Fattorini, A.V. Fursikov and others.

The following result is get in this work

**Theorem 1.** *Let $\theta \in W_2^2(-\infty,\infty)$, $\theta(t) = 0$ with $t \leq 0$ and $\|\theta(t)\|_{W_2^2(R_+)} \leq M$. Then there is $\mu(t)$ control that ensures the fulfillment of condition (4).*

Note that the similar results hold for the pseudo - parabolic equations and are easily generalized for the spatial case.

**THE BOUNDARY-INTERNAL PROBLEM FOR A SYSTEM OF THE SECOND-ORDER EQUATIONS OF A MIXED TYPE**

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In this paper, we study a problem with boundary and data inside the domain of regularity of the solution for a system of partial differential equations of a mixed type. The problem under study belongs to the class of inverse and incorrectly posed problems of mathematical physics. The main problem is the proof of the conditional correctness of the above problem, namely the proof of the uniqueness of the solution and its conditional stability on the set of correctness.

Let a pair of functions $(v(x,t), u(x,t))$ be a solution of the equation

$$\begin{cases} sgnx v_{tt}(x,t) + v_{xx}(x,t) = sgnx f(x,t), \\ sgnx u_{tt}(x,t) + u_{xx}(x,t) = sgnx v(x,t), \end{cases}$$

in the field $D = \{-1 < x < 1,\ x \neq 0,\ 0 < t < T\}$ and satisfies a primary and internal

$$v\mid_{t=0} = \varphi_1(x),\ v\mid_{t=t_1} = \varphi_2(x),\ u\mid_{t=t_2} = \varphi_3(x),\ u\mid_{t=t_3} = \varphi_4(x),$$

boundary

$$v(-1,t) = v(1,t) = 0, \quad 0 \le t \le T, u(-1,t) = u(1,t) = 0, \quad 0 \le t \le T,$$

as well as gluing conditions

$$v(-0,t) = v(+0,t),\ v_x(-0,t) = v_x(+0,t),\ 0 \le t \le T,$$

$$u(-0,t) = u(+0,t),\ u_x(-0,t) = u_x(+0,t), \quad 0 \le t \le T.$$

A number of uniqueness and stability theorems for solving internal problems of elliptic equations are obtained in the works of M. A. Lavrentiev and M.M. Lavrentiev, S.P. Shishatsky, A. Abdukarimov. Internal problems for parabolic equations are considered in the works of M.M. Lavrentev, B.K.Amonov, S.P. Shishatsky et al. In this paper, we prove theorems concerning the uniqueness and conditional stability of the investigated problem.

# NONLOCAL BOUNDARY VALUE PROBLEMS FOR THE SECOND ORDER MIXED TYPE DIFFERENTIAL EQUATION

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In this paper, we study problems of the second order mixed type differential equation with partial derivatives with non-local conditions defined as a linear combination of the values of the desired function and its derivatives at various points of the boundary.

Nonlocal problems of various classes of equations were considered in the works of A.A. Dezin, V.A. Ilyin, M.A. Naimark, E.I. Moiseev, K.B. Sabitov, S.G. Krain and G.I. Laptev, O.A. Repin, I.E. Egorov, A.I. Kozhanov, S.P.Shishatsky, N.I. Ptashnik and others.

The investigated problem, generally speaking, is related to the class of incorrectly posed problems of mathematical physics, namely, in this problem there is no continuous dependence of the solution on the initial data. In addition, the problem of small denominators arises in these problems, i.e. not for all data, the solution to the problem exists and it is unique.

In this paper, we prove theorems of conditional stability on a set of correctness.

Statement of the problem. Let $\Omega$ be a finite interval $(-1,\ 1)$ of the axis $Ox$, $Q$ is a rectangle $\Omega \times (0,\ T)$, $0 < T < +\infty$. Consider the boundary problem: find the function $u(x,t)$ which is solution to the equation

$$sgnx\, u_{tt}(x,t) + u_{xx}(x,t) = 0$$

on the $Q$ and satisfies the following conditions:
the nonlocal

$$\left.\begin{array}{l} \alpha_1 u(x,0) + \beta_1 u(x,T) = f(x), \\ \alpha_2 u_t(x,0) + \beta_2 u_t(x,T) = g(x) \end{array}\right\},\ |\alpha_i| + |\beta_i| \neq 0,\ i = 1, 2.,\ x \in [-1,\ 1],$$

boundary

$$u(-1,t) = u(1,t) = 0,\ \ t \in [0,\ T],$$

and gluing conditions

$$u(-0,t) = u(+0,t),\ u_x(-0,t) = u_x(+0,t),\ t \in [0,\ T],$$

where $f(x),\ g(x)$ are sufficient smooth functions.

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## BOUNDARY VALUE PROBLEM FOR SECOND ORDER MIXED TYPE NONHOMOGENEOUS DIFFERENTIAL EQUATION WITH TWO DEGENERATE LINES

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In this paper, we consider ill-posed boundary value problem for a second order nonhomogeneous differential equation with two degenerate lines.

Let $u(x, y, t)$ is solution of the equation

$$u_{tt}(x, y, t) = sign(x)u_{xx}(x, y, t) + sign(y)u_{yy}(x, y, t) + f(x, y, t)$$

on the region $\Omega = \{(x, y, t)|(-1; 1)^2 \times (0; T), T < \infty,\ x \neq 0,\ y \neq 0\}$.

**Problem.** Find the function $u(x, y, t)$ satisfying equation (1) and the following conditions: initial

$$\left.\frac{\partial^i u(x, y, t)}{\partial t^i}\right|_{t=0} = \varphi_{i+1}(x, y), i = 0, 1; \Pi = \left\{(x, y)|\,(-1; 1)^2\right\},$$

boundary

$$u(x, y, t)\Big|_{\substack{x=-1\\x=+1}} = 0,\ \ (y, t) \in [-1; 1] \times [0; T],$$
$$u(x, y, t)\Big|_{\substack{y=-1\\y=+1}} = 0,\ \ (x, t) \in [-1; 1] \times [0; T],$$

and gluing conditions

$$\left.\frac{\partial^i u(x, y, t)}{\partial x^i}\right|_{x=-0} = \left.\frac{\partial^i u(x, y, t)}{\partial x^i}\right|_{x=+0}, (y, t) \in [-1; 1] \times [0; T],$$
$$\left.\frac{\partial^i u(x, y, t)}{\partial y^i}\right|_{y=-0} = \left.\frac{\partial^i u(x, y, t)}{\partial y^i}\right|_{y=+0}, (x, t) \in [-1; 1] \times [0; T].$$

where $i = 0, 1$; $\varphi_1(x, y)$, $\varphi_2(x, y)$ and $f(x, y, t)-$ are given sufficiently smooth functions.

The study problem for mixed type equations with various kind of change or degeneration has been the object of study by many researchers.

In this paper, problem (1) - (4) is investigated for conditional correctness, namely, theorems on uniqueness and conditional stability are proved.

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## A TWO-COMPONENT DEEP BED FILTRATION MODEL WITH MULTISTAGE DEPOSITION KINETICS

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When the suspension flows through the porous medium, some of the particles, under the influence of a number of forces, may come into contact with the filter grains and deposit on the surface of the grains or on previously deposited particles. Consequently, the geometry and the structure of the medium, and the surface characteristics of the filter grains may be modified significantly. These changes, in turn, affect the flow of the suspension through the medium and the deposition. Deep-bed filtration usually viewed on microscopic and macroscopic levels [1]. Macroscopically, filtration process may be described as a system of phenomenological differential equations for particle mass balance, kinetics of particle deposition, the Darcy's law and Carmeny-Kozen equation[2]

$$m_0\frac{\partial c^{(i)}}{\partial t}+v\frac{\partial c^{(i)}}{\partial x}+\frac{\partial \rho_a^{(i)}}{\partial t}+\frac{\partial \rho_p^{(i)}}{\partial t}=0, \tag{1}$$

$$\frac{\partial \rho_p^{(i)}}{\partial t}=\beta_p^{(i)}\left(\rho_p^{(1)},\,\rho_p^{(2)}\right)c^{(i)}, \tag{2}$$

$$\frac{\partial \rho_a^{(i)}}{\partial t}=\frac{\beta_a^{(i)}}{1+\gamma^{(i)}\left|\nabla p\right|}c^{(i)}-\beta_a^{(i)}\frac{\rho_a^{(i)}\left(1+\omega^{(i)}\left|\nabla p\right|\right)}{\rho_{a0}^{(i)}}c_0^{(i)}, \tag{3}$$

$$v=k(m)\left|\nabla p\right|, \tag{4}$$

$$k(m)=\frac{k_0m^3}{(1-m)^3}, \tag{5}$$

$$m=m_0-(\rho_a^{(1)}+\rho_a^{(2)}+\rho_p^{(1)}+\rho_p^{(2)}), \tag{6}$$

where $c^{(i)}$-concentration of suspension, $v$ - filtration velocity, $m$, $m_0$ - the current and initial porosity of the media, $\beta_a^{(i)}$, $\beta_p^{(i)}$- the coefficients characterizing the kinetics in the active zone and passive zones respectively, $\rho_a^{(i)}$, $\rho_p^{(i)}$ - concentration of deposition formed in active zones and passive zones respectively, $|\nabla p|$ - the modul of the gradient of pressure, $k(m)$ - coefficient of permeability, $i=1,2$ - correspond to the component numbers.

Equations (1) - (4) with (5), (6) and given initial and boundary conditions are solved by the method of finite differences. The multi-stage nature of the deposition kinetics can lead to various effects that are not characteristic for the transport of one-component suspensions with a one-stage particle deposition kinetics. In particular, in distribution of the concentration of suspended particles in a moving fluid non-monotonic dynamics are obtained at given points of the medium.

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## THE STEIN-TIKHOMIROV METHOD AND NONCLASSICAL CLT

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Suppose that $F(x)$is an arbitrary distribution function and

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{u^2}{2}} du$$

is a distribution function of the standard normal random variable (r.v.). In [1] Stein proposed an universal method for estimating the quantity

$$\delta = \sup_x |F(x) - \Phi(x)| \, .$$

We will modify Stein's method in terms of characteristic functions (ch.f.).

Consider a class of ch.f.

$$F = \left\{ f(t) : \, f'(0) = 0, \, \sigma^2 = -f''(0) < \infty \right\}$$

and define the Stein-Tikhomirov (S.-T.) operator

$$\Delta f(t) = f'(t) + \sigma^2 t f(t).$$

Let

$$X_{n1}, \, X_{n2}, \, ..., \, n = 1, \, 2, \, ...$$

be an array of r.v.. Assume that

$$EX_{nj} = 0, \; \sigma_{nj}^2 = EX_{ni}^2, \; j = 1, \, 2, \, ..., \; \sum_{j=1}^{\infty} \sigma_{nj} = 1$$

and denote

$$S_n = X_{n1} + X_{n2} + \ldots \, , \; F_n(x) = P\left(S_n < x\right), \, n = 1, \, 2, \, ....$$

**Theorem.** *The following convergence holds*

$$\sup_x |F_n(x) - \Phi(x)| \to 0, \; n \to \infty \, ,$$

*if and only if for any* $T > 0$

$$\sup_{|t| \le T} \sum_j |\Delta\left(f_{nj}(t)\right)| \to 0 \, ,$$

*where* $f_{nj}(t)$ *is the ch.f. corresponding to the distribution function* $F_{nj}(x)$.

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## APPROXIMATIONS OF DISTRIBUTIONS WITH A GENERALIZED POISSON DISTRIBUTION

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In the modern theory of summation of independent random variables (rv), along with the central limit theorem (approximation of the distribution of sums of independent rv by Gaussian distribution), limiting theorems on convergence to generalized Poisson distributions play an important role.

Let be

$$X_{n1}, \ldots X_{nn}, \ldots \qquad n = 1, 2, \ldots$$

a sequence of series of independent r.v. We assume that

$$S_n = X_{n1} + \ldots + X_{nn}, \text{ and } EX_{nj} = 0,\ EX_{nj}^2 = DX_{nj} = \lambda_{nj},\ j = 1, 2 \ldots$$

$$\sum_{j=1}^{n} \lambda_{nj} = DS_n \to \lambda \qquad n \to \infty. \tag{1}$$

Put at j=1,2,...

$$F_{nj}(x) = P(X_{nj} < x), \qquad x \in \mathbb{R}.$$

Then for the distribution of the sum $S_n$ there is equality

$$F_n(x) = P(S_n < x) = F_{n1} * F_{n2} * \ldots * F_{nn}.$$

The generalized Poisson distribution law with mean expectation $\alpha$ and variance $\lambda > 0$, i.e. law with a characteristic function (ch.f.)

$$\pi(\alpha, \lambda, t) = exp\{i\alpha t + \lambda(e^{it} - 1 - it)\},$$

we will denote $G(\alpha, \lambda, x)$ . In the following, the following distribution functions (d.f.) are considered

$$G_{nj}(x) = G(0, \lambda_{nj}, x), \quad j = 1, 2, \qquad ,$$

$$G_\lambda(x) = G(0,\lambda,x) = G_{n1} * G_{n2} * \ldots = \prod_{j=1}^{\infty} * G_{nj}.$$

Therefore $G_\lambda(x)$ denotes the d.f. of the Poisson law with the parameter

$$\lambda = \sum_{j=1}^{\infty} \lambda_{nj} = \lim_{n\to\infty} DS_n.$$

In studies on limit theorems, the ratio for the distributions of sums of independent r.v.

$$L_n(\varepsilon) = \sum_{j=1}^{n} \int_{|x-1|>\varepsilon} x^2 dF_{nj}(x) \to 0, \quad n \to \infty. \qquad (L)$$

(L) valid for any $\varepsilon > 0$, is called the Lindeberg condition.

Within the limits of the introduced notation and conditions, the following results are valid:

**Theorem 1.** *Let the Lindeberg conditions (L) and (1) be fulfilled. Then*

$$F_n(x) = P(S_n < x) \Longrightarrow G_\lambda(x) = \sum_{k<x+\lambda} e^{-\lambda} \cdot \frac{\lambda^k}{k!}, \qquad (2)$$

*where* $\Longrightarrow$ *denotes the weak convergence of the distributions.*

We introduce the value $\alpha > 0$,

$$M_n(\alpha) = \sum_{j=1}^{n} \int_{|x-1|\geq|x|\geq 1} |x|^{2+\alpha} dF_{nj}(x) + \sum_{j=1}^{n} \int_{|x-1|>1} x^2 dF_{nj}(x) = m_n(\alpha) + L_n(1).$$

It is easy to notice that if at $\alpha = \alpha_0 > 0$, $m_n(\alpha_0) \to 0$ then $m_n(\alpha) \to 0, n \to \infty$ is for any $\alpha > 0$.

Taking into account the above remarks, it will say that the condition is fulfilled, if with some $\alpha > 0$

$$M_n(\alpha) \to 0, \quad n \to \infty \qquad (M_\alpha).$$

Therefore, in the relation $(M_\alpha)$, without restricting generality, we can put $\alpha = 1$ i.e we will assume that the condition

$$M_n(1) = m_n(1) + L_n(1) = \sum_{j=1}^{n} \int_{|x-1|\geq|x|\geq 1} |x|^3 dF_{nj}(x) + L_n(1) \to 0, \quad n \to \infty.$$

is fulfilled.

**Theorem 2.** *There is an equivalence relation*

$$M_n(1) \Longleftrightarrow (L)(M_n(\alpha) \to 0, \quad \text{for some } \alpha > 0) \Longleftrightarrow (L)$$

**Theorem 3.** *If condition* $(M_1)$, *is satisfied, then the limit relation (2) holds.*

Theorems 1-3 generalize and clarify the limit theorems on convergence to the Poisson distribution law, given in the monograph [1] (Chapter IX).

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## ON THE "Φ-QUICKLY" CONVERGENCE OF THE SEQUENCE OF SUMS OF ORDER STATISTICS

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This paper is a continuation of the author's article where the concept of "Φ-quickly" convergence of a sequence of random variables (s.r.v.) is introduced.

Let $\{\zeta_n, n \geq 1\}$-s.r.v., $\forall\varepsilon > 0, \chi(\varepsilon) = \sup\{n \geq 1, |\zeta_n| > \varepsilon\}$ and for some non-negative function $\varphi(x)$, defined on $[1, \infty)$, we put

$$\Phi(x) = \int_1^x \varphi(t)dt.$$

**Definition.** [1] S.r.v. $\{\zeta_n, n \geq 1\}$ at $n \to \infty$ "Φ-quickly" converges to zero, if $\mathrm{E}\Phi(\chi(\varepsilon)) < \infty, \forall\varepsilon > 0$.

This definition is one of the variants of the generalization of the "r-quickly" convergence of a s.r.v. introduced in [2]. Many tasks based on this definition cover a wide range of problems, in particular, with the summation of r.v., having different natures of interrelationship [1].

In this work, $n \to \infty$ investigates the behavior of the sum

$$\zeta_n(l) = \frac{1}{H(n)} \sum_{k=l+1}^{n} \xi_n^{(k)},$$

where $\xi_n^{(k)}, 1 \leq k \leq n-$ order statistics, induced by independent, identically distributed r.v. $\xi_1, \xi_2, ..., \xi_n$ arranged in descending order, and $H(x)$- defined on $[1, \infty)$ positive strictly increasing function. Assuming that $\forall\varepsilon > 0, \chi(\varepsilon, l) = \sup\{n \geq l, |\zeta_n(l)| > \varepsilon\}\}$

1. We described the class of distribution functions $\xi_1$ and classes of functions $\varphi(x)$ and $H(x)$, in which at fixed

$$l \geq 0 \text{ and } \forall\varepsilon > 0, E\Phi(\chi[(\varepsilon, l)] < \infty$$

2. Other definitions generalizing of "r-quickly" convergence are given. The conditions of equivalence of these definitions are established.

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## POSITIVE FIXED POINTS OF LYAPUNOV'S OPERATOR

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The study of nonlinear integral operators is used to solve current problems of theoretical physics. In particular, solving of nonlinear (quadratic) integral equations is used to find Gibbs measurements. In the work [1] is studied quadratic integral equations. This work is devoted to analyzing positive fixed points of Lyapunov's operator.

Let $\varphi_1(t)$, $\varphi_2(s)$ and $\varphi_3(u)$ are positive function from $C_0^+[0,1]$. We consider Lyapunov's operator $A$:

$$(Af)(t) = \int_0^1 \int_0^1 (\varphi_1(t) + \varphi_2(s) + \varphi_3(u))\, f(s)f(u)dsdu$$

and quadratic operator $P$ on $\mathbb{R}^3$ by the rule

$$P(x,y,z) = (\alpha_{11}x^2 + xy + xz,\ \alpha_{21}x^2 + \alpha_{22}xy + \alpha_{22}xz,\ \alpha_{31}x^2 + \alpha_{33}xy + \alpha_{33}xz),$$

here

$$\alpha_{11} = \int_0^1 \varphi_1(s)ds > 0;$$

$$\alpha_{22} = \int_0^1 \varphi_2(s)ds > 0, \quad \alpha_{21} = \int_0^1 \varphi_1(s)\varphi_2(s)ds > 0;$$

$$\alpha_{33} = \int_0^1 \varphi_3(s)ds > 0, \quad \alpha_{31} = \int_0^1 \varphi_1(s)\varphi_3(s)ds > 0.$$

By the theorem [2], we well know, that there is a fixed point of Lyapunov's operator $A$.

**Lemma 1.** *The Lyapunov's operator $A$ has a nontrivial positive fixed point iff the quadratic operator $P$ has a nontrivial positive fixed point, morever $N_{fix}^+(A) = N_{fix}^+(P)$.*

We define quadratic operator (QO) $\mathcal{Q}$ in cone $\mathbb{R}_3$ by the rule

$$\mathcal{Q}(x,y,z) = (a_{11}x^2 + xy + xz,\ a_{21}x^2 + a_{22}xy + a_{22}xz,\ a_{31}x^2 + a_{33}xy + a_{33}xz).$$

**Lemma 2.** *If the point $\omega = (x_0, y_0, z_0) \in \mathbb{R}_2^+$ is fixed point of QO $\mathcal{Q}$, then $x_0$ is a root of the quadratic algebraic equation*

$$(a_{21} + a_{31} - a_{11}a_{22} - a_{11}a_{33})x^2 + (a_{11} + a_{22} + a_{33})x - 1 = 0. \tag{1}$$

**Lemma 3.** *If the positive number $x_0$ is root of the quadratic algebraic Eq.(1), then the point $\omega_0 = (x_0,\ x_0(a_{21}x_0 + a_{22}(1 - a_{11}x_0)),\ x_0(a_{31}x_0 + a_{33}(1 - a_{11}x_0)))$ is fixed point of QO $\mathcal{Q}$.*

**Theorem 1.** *QO $\mathcal{Q}$ has a unique nontrivial positive fixed point.*

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## TRANSVERSALITY OF VOLTERRA TYPE OPERATORS ACTING IN THE SIMPLEX $S^{m-1}$

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A Volterra stochastic operator is defined by the equalities

$$x'_k = x_k\left(1+\sum_{i=1}^{m} a_{ki}x_i\right), \quad k=\overline{1,m}, \tag{1}$$

where $x=(x_1,\ldots,x_m)\in S^{m-1}$, $A=(a_{ki})$ is a skew-symmetric matrix, and $|a_{ki}|\leq 1$. It is known that $V: S^{m-1}\to S^{m-1}$ is a homeomorphism.

**Definition 1.** A skew-symmetric matrix $A$ is said to be transversal if any major minor of even order is positive.

If $A$ is transversal, then the operator $V$ is also called transversal.

**Theorem 1.** *The set of all transversal Volterra operators is a massive subset in the set of all Volterra operators.*

**Theorem 2.** *1) The set of fixed points of transversal Volterra operators is finite and nonempty.*
*2) Any fixed point has only odd number of nonzero coordinates.*

**Problem.** *Let $k\geq 3$. Does the existence of a fixed point with $2k+1$ nonzero coordinates always imply the existence of a fixed point with $2k-1$ nonzero coordinates?*

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# QUADRATIC STOCHASTIC OPERATORS WITH HOMOGENEOUS TOURNAMENT

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Let $A = (a_{ki})$, $k, i = \overline{1, m}$ is a skew-symmetric matrix $A' = -A$, where $|a_{ki}| \leq 1$.

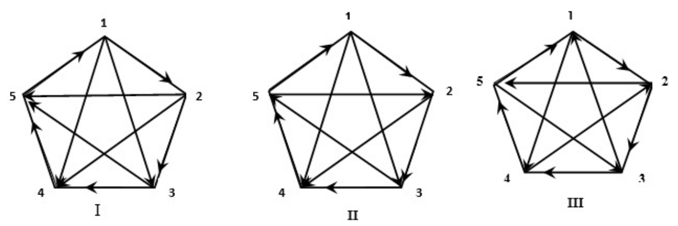
**Definition 1.** A quadratic stochastic operator $V : S^{m-1} \to S^{m-1}$ is defined by

$$V : x'_k = x_k \left( 1 + \sum_{i=1}^{m} a_{ki} x_i \right), \quad k = \overline{1, m}, \tag{1}$$

where $S^{m-1} = \{x = (x_1, ...x_m) \in R^m : x_i \geq 0 \sum\limits_{i=1}^{m} x_i = 1\}\}$ $(m-1)$-dimensional simplex, is called the Lotka-Volterra operator. Let $a_{ki} \neq 0$ for $k \neq i$. Along with the dynamical system (1) we consider the complete graph $G_m$ with $m$ vertices. Let us specify a direction on the edges of $G_m$ as follows:the edge joining vertices $k$ and $i$ is directed from the $k$-th to the $i$-th vertex if $a_{ki} < 0$, and has the opposite direction if $a_{ki} > 0$. The directed graph thus obtained is called a tournament and denoted by $T_m$. A tournament is called strong if there is a Hamilton contour passing through all the top. A tournament is called transitive if it does not contain a cycle of length 3.

**Definition 2.** A tournament $T_m$ is called homogeneous if any subtournament is either strong or transitive.

**Proposition 1.** *At* $m = 5$ *there exist* 12 *pairwise nonisomorphic tournaments and* 6 *of them strong tournaments. Three of the previous presented strong tournaments are homogeneous.*




**Proposition 2.** *At* $m = 6$ *there exist* 56 *pairwise nonisomorphic tournaments. Four these tournaments are homogeneous.*

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# THE DESCRIPTION OF 12-DIMENSIONAL SOLVABLE LIE ALGEBRAS WHOSE NILRADICAL HAS CHARACTERISTIC SEQUENCE $C(L) = (6, 3, 1)$

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It is well-known that solvable Lie algebras play a crucial role in the structure theory of finite-dimensional complex Lie algebras. Description of solvable Lie algebras can be obtained be means of nilradical and its derivations. In [1] and [2] the solvable Lie algebras which nilradical is a model algebra with characteristic sequence $C(L) = (n_1, n_2, \dots, n_s, 1)$ and maximal dimension of complementary space (i.e., $q = s$) is obtained.

In this work we consider the case of $q = s - 1$ and give the description of 12-dimensional solvable Lie algebras whose nilradical has characteristic sequence equal to $C(L) = (6, 3, 1)$.

**Definition 1.** A vector space with bilinear a bracket $(L, [-, -])$ over a field $\mathbb{F}$ is called a Lie algebra if for any $x, y, z \in L$ the following identities hold:

1. $[x, y] = -[y, x]$ antisymmetry identity,
2. $[x, [y, z]] + [y, [z, x]] + [z, [x, y]] = 0$ Jacobi identity.

For a given Lie algebra $(L, [-, -])$ the lower central and the derived series are defined recursively as follows:

$$L^1 = L,\ L^{k+1} = [L^k, L],\ k \geq 1, \qquad L^{[1]} = L,\ L^{[s+1]} = [L^{[s]}, L^{[s]}],\ s \geq 1.$$

**Definition 2.** An algebra $L$ is said to be solvable (respectively, nilpotent) if there exists $s \in N$ (respectively, $m \in N$) such that $L^s = 0$ ($L^{[m]} = 0$).

The maximal nilpotent ideal of an algebra is called a nilradical.

Let $x$ be a nilpotent element of the set $L \setminus L^2$. For the nilpotent operator of rigth multiplication $R_x$ we define a decreasing sequence $C(x) = (n_1, n_2, \dots, n_k)$, which consists of the dimensions of the Jordan blocks of the operator $R_x$. In the set of such sequences we consider the lexicographic order, that is, $C(x) = (n_1, n_2, \dots, n_k) \leq C(y) = (m_1, m_2, \dots, m_s)$ iff there exists $i \in \mathbb{N}$ such that $n_j = m_j$ for any $j < i$ and $n_i < m_i$.

**Definition 3.** The sequence $C(L) = maxC(x)_{x \in L \setminus L^2}$ is called the characteristic sequence of the algebra $L$.

Consider the nilpotent Lie algebra with characteristic sequence $C(L) = (6, 3, 1)$ and the following table of multiplications:

$$N_{6,3,1} : \begin{cases} [e_i, e_1] = -[e_1, e_i] = e_{i+1}, & 2 \leq i \leq 6, \\ [f_i, e_1] = -[e_1, f_i] = f_{i+1}, & 1 \leq i \leq 2 \end{cases}$$

where $\{e_1, e_2, e_3, e_4, e_5, e_6, e_7, f_1, f_2, f_3\}$ is a basis of the algebra.

In the following theorem we give the description of $12-$ dimensional solvable Lie algebra with nilradical is $N_{6,3,1}$.

**Theorem 1.** *Let $L$ be a 12 dimensional solvable Lie algebra with nilradical $N_{6,3,1}$, then*

*L is isomorphic to one of the following algebras:*

$$R_1: \begin{cases} [e_i, e_1] = -[e_1, e_i] = e_{i+1}, \quad 2 \le i \le 6, \\ [f_i, e_1] = -[e_1, f_i] = f_{i+1}, \quad 1 \le i \le 2, \\ [e_1, x] = -[x, e_1] = e_1, \\ [e_i, x] = -[x, e_i] = (i-2)e_i, \quad 3 \le i \le 7, \\ [f_i, x] = -[x, f_i] = (i-1)f_i, \quad 2 \le i \le 3, \\ [e_i, y] = -[y, e_i] = e_i, \quad 2 \le i \le 7, \end{cases}$$

$$R_2: \begin{cases} [e_i, e_1] = -[e_1, e_i] = e_{i+1}, \quad 2 \le i \le 6, \\ [f_i, e_1] = -[e_1, f_i] = f_{i+1}, \quad 1 \le i \le 2, \\ [e_1, x] = -[x, e_1] = e_1 + \alpha e_2 + \beta f_1, \\ [e_i, x] = -[x, e_i] = (i-2)e_i, \quad 3 \le i \le 7, \\ [f_i, x] = -[x, f_i] = (i-1)f_i, \quad 2 \le i \le 3, \\ [e_1, y] = -[y, e_1] = \delta f_1, \\ [e_i, y] = -[y, e_i] = \varepsilon f_{i-1}, \quad 2 \le i \le 4, \\ [f_i, y] = -[y, f_i] = f_i, \quad 1 \le i \le 3, \\ [x, y] = \eta e_7 + \mu f_3, \end{cases}$$

*where* $\delta, \varepsilon \in \{0, 1\}$,

$$R_3: \begin{cases} [e_i, e_1] = -[e_1, e_i] = e_{i+1}, \quad 2 \le i \le 6, \\ [f_i, e_1] = -[e_1, f_i] = f_{i+1}, \quad 1 \le i \le 2, \\ [e_1, x] = -[x, e_1] = \alpha e_2 + \beta f_1, \\ [e_j, x] = -[x, e_j] = e_j + \sum\limits_{i=4}^{9-j} \alpha_i e_{i+j-2} + \sum\limits_{i=1}^{5-j} \beta_i f_{i+j-2}, \quad 2 \le j \le 7, \\ [f_1, x] = -[x, f_1] = \gamma_1 f_2 + \gamma_2 f_3, \\ [f_2, x] = -[x, f_2] = \gamma_1 f_3, \\ [e_1, y] = -[y, e_1] = \alpha_1 e_2 + \beta_2 f_1, \\ [e_j, y] = -[y, e_j] = \sum\limits_{i=4}^{9-j} \sigma_i e_{i+j-2} + \sum\limits_{i=1}^{5-j} \varsigma_i f_{i+j-2}, \ 2 \le j \le 5, \\ [f_j, y] = -[y, f_j] = f_j + \xi_1 f_{j+1} + \lambda_1 f_{j+2}, \quad 1 \le j \le 3, \\ [x, y] = \sum\limits_{i=3}^{7} \eta_i e_i + \sum\limits_{i=1}^{3} \mu_i f_i. \end{cases}$$

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## PURSUIT-EVASION GAME ON THE ICOSIDODECAHEDRON IN THE SPACE $\mathbb{R}^3$

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Let $M_{30}$ denote the graph of 1-skeleton of the icosidodecahedron with 30 vertices in Euclidian space $\mathbb{R}^3$ [1-3]. The team of Pursuers $\mathbf{P} = \{P_1, P_2, \ldots, P_n\}$ and one Evader $Q$ moving along $M_{30}$ play pursuit-evasion game. All players have equal maximal speeds. Let $\mathbf{P(t)} = \{P_1(t), P_2(t), \ldots, P_n(t)\}$ and $Q(t)$ be points of player's positions at the moment $t \geq 0$. The aim of the team of Pursuers is to reach the equality $P_i(t) = Q(t)$ for some $i = 1, 2, \ldots, n, t, t \geq 0$ for any of initial positions of players. The aim of Evader is opposite to pursuer's aim i.e. choosing some initial position to hold the condition $P_i(t) \neq Q(t)$ for all $i = 1, 2, \ldots, n$ and $t, t \geq 0$ (for details of see [4-5]). Obviously, if $n$ is great enough then the team of Pursuers can win the game. The least value of $n$, that $n$ Pursuers win the game will be denote by $N(M_{30})$.

**Theorem 1.** *$N(M_{30}) = 3$ for the icosidodecahedron $M_{30}$ in Euclidian space $\mathbb{R}^3$.*

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## AN ANALOGUE OF THE KYTMANOV'S THEOREM FOR $A(z)$-ANALYTIC FUNCTIONS

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Let $D \subset C$ be a convex domain. The functions identified in this area should look at the real-analytical solution of the Beltrami equation

$$f_{\bar{z}}(z) = A(z) f_z(z) \tag{1}$$

The function $A(z)$ is, in general, assumed to be measurable with $|A(z)| \leq c < 1$ almost everywhere in the domain $D$. Solutions of equation (1) are often referred to as $A(z)$-*analytical functions* in [4].

The function $f(z)$ belongs to the Hardy class $H^p(L(a;R))$, $p \geq 1$, if the function $f(z)$ is regular and bounded in the set $L(a;R) = \left\{ \mid \psi(z;a) \mid = \mid z - a + \overline{\int\limits_{\gamma(z;\zeta)} \overline{A(\tau)} d\tau} \mid < R \right\}$. [3]

$$\exists M > 0, 0 < r < R, \int\limits_{|z|=r} \mid f(z) \mid^p dz \leq M.$$

Hardy $H^p$ space with $0 < p < \infty$ is the class of functions in the open set of $L(a;R) = \{\mid \psi(z;a) \mid < R\}$

$$f \mid_{H^p} = \sup_{0<r<R} \left| \frac{1}{2\pi} \int\limits_{|z|=r} \mid f(z) \mid^p dz \right|^{\frac{1}{p}} < \infty.$$

We denote $f \in H^p_A(D)$, if $f \in O_A(L(a;R))$ and $f \in H^p(L(a;R)$.

The $L^\infty(D)$ is constructed from the space of $\mathcal{L}^\infty(D, F, \mu)$ of measurable functions, bounded almost everywhere, by identifying with each other functions differing only in the set of zero measure and possible definition:

$$|| f ||_\infty = ess \sup_{z \in D} \mid f(z) \mid = vrai \sup_{z \in D} \mid f(z) \mid,$$

where $ess \sup$ or $vrai \sup$ is an essential supremum of function that is, the $f : D \longrightarrow C$ function is the lower bound of the set of such numbers $c$, that $|f(z)| \leq c, z \in D$ almost the entire area.

We denote $f \in L^\infty(D)$, if $f \in O_A(L(a;R))$ and $f \in L^\infty(L(a;R)$. Let the non-discriminatory visible mapping of the $\omega(\zeta)$ to automorphisms of $L(a;R)$ lemniscates. Let $M$ be the set of positive Lebesgue measure on the $\partial L(a;R)$ lemniscate. Consider $a$ fixed point $b \in L(a;R)$ and the images of the $\omega(M) = M_b$ of the set $M$ under automorphisms of $\omega(\zeta)$ lemniscates.

Suppose that for each $\omega(M)$ there is a sequence of the function $\varphi^b_m = \varphi^{M_a}_b \in L^\infty_A(M_b)$ such that for any $f \in H^1_A(L(a;R))$

$$f(a) = \lim_{m \to \infty} \frac{1}{2\pi i} \int\limits_{M_b} f(\zeta) \varphi^b_m(\zeta) \frac{d\zeta + A d\bar{\zeta}}{\zeta - a + \overline{\int\limits_{\gamma(a;\zeta)} \overline{A(\tau)} d\tau}}. \tag{2}$$

If $A$ is antianalytic in $L(a;R)$, then the following statement holds.

**Theorem** (analogue of the Kytmanov's theorem). *If the $f \in H^1_A(L(a;R))$ set is the $M \subset \partial L(a;r)$ of positive Lebesgue measure, then the formula*

$$f(b) = \lim_{m \to \infty} \frac{1}{2\pi i} \int\limits_{M} f(\zeta) \varphi^b_m(\omega(\zeta)) \frac{d\zeta + A d\bar{\zeta}}{\zeta - b + \overline{\int\limits_{\gamma(b;\zeta)} \overline{A(\tau)} d\tau}} \tag{3}$$

*is true for any point $b \in L(a;r)$.*

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## INDIVIDUAL ERGODIC THEOREM IN LORENTZ SPACES

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The classical individual ergodic theorem for the $L_p$-spaces $L_p(\Omega, \mathcal{A}, \mu)$, $1 \le p < \infty$, states that for any absolute linear contraction $T : L_p(\Omega, \mathcal{A}, \mu) \to L_p(\Omega, \mathcal{A}, \mu)$ and for any function $f \in L_p(\Omega, \mathcal{A}, \mu)$, there exists a function $\widehat{f} \in L_p(\Omega, \mathcal{A}, \mu)$ such that the averages $A_n(T)(f) = \frac{1}{n+1} \sum_{k=0}^{n} T^k(f)$ converge almost everywhere to $\widehat{f}$ (see, for example, [1, Ch. VIII §5 ]). A variant of individual ergodic theorem for Orlicz spaces established in [2]. In this note we give a strengthened version of the individual ergodic theorem for the Lorentz spaces.

Let $(\Omega, \mathcal{A}, \mu)$ be a measure space with a complete $\sigma$-finite measure. By $L(\Omega)$ we denote the algebra of all classes of almost everywhere equal real measurable functions on $(\Omega, \mathcal{A}, \mu)$. Let $L_\mu(\Omega)$ be the subalgebra in $L(\Omega)$ of all functions $f \in L(\Omega)$, for which $\mu(\{|f| > \lambda\}) < \infty$ for some $\lambda = \lambda(f) > 0$. If $f \in L_\mu(\Omega)$, then a non-increasing rearrangement $f^*(t)$, $t > 0$, is determined by the equation $f^*(t) = \inf\{\lambda > 0 : \mu(\{|f| > \lambda\}) \le t\}$.

Let $\psi$ be a concave function on $[0, \infty)$, $\psi(0) = 0$ and $\psi(t) > 0$ for all $t > 0$, and let

$$\Lambda_\psi(\Omega, \mathcal{A}, \mu) = \left\{ f \in L_\mu(\Omega) : \ \|f\|_\psi = \int_0^\infty f^*(t) d\psi(t) < \infty \right\}$$

corresponding Lorentz space. It is well known that $(\Lambda_\psi(\Omega, \mathcal{A}, \mu), \|\cdot\|_\psi)$ is a Banach space (see, for example, [3, Chapter II. §5]).

Linear operator $T : L_1(\Omega) \to L_1(\Omega)$ is called an absolute contraction if $\|T(f)\|_1 \le \|f\|_1$ for all $f \in L_1(\Omega, \mathcal{A}, \mu)$ and $\|T(f)\|_\infty \le \|f\|_\infty$ for all $f \in L_1(\Omega, \mathcal{A}, \mu) \cap L_\infty(\Omega, \mathcal{A}, \mu)$. It is known that each absolute contraction $T$ extends to linear contraction of Lorentz space $(\Lambda_\psi(\Omega, \mathcal{A}, \mu), \|\cdot\|_\psi)$, which is also denoted by $T$.

According to Egorov's theorem, in the case $\mu(\Omega) < \infty$, almost everywhere convergence coincides with an almost uniform convergence. Therefore, the classical individual ergodic theorem for $L_p$-spaces states that in the case $\mu(\Omega) < \infty$, the averages $A_n(T)(f)$ converge almost uniformly. For non-finite measure almost uniform convergence (in the Egorov's sense) is usually stronger than the almost everywhere convergence.

The following theorem is strengthened version of individual ergodic theorem for Lorentz spaces.

**Theorem 1.** Let $(\Omega, \mathcal{A}, \mu)$ be a measure space with a complete $\sigma$-finite measure and let $T : \Lambda_\psi(\Omega, \mathcal{A}, \mu) \to \Lambda_\psi(\Omega, \mathcal{A}, \mu)$ be an arbitrary absolute contraction. Then, for every $f \in \Lambda_\psi(\Omega, \mathcal{A}, \mu)$ there exists a function $\widehat{f} \in \Lambda_\psi(\Omega, \mathcal{A}, \mu)$ such that the averages $A_n(T)(f)$ converge almost uniformly to the function $\widehat{f}$ at $n \to \infty$.

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## SOME STRONGLY NILPOTENT FILIFORM LEIBNIZ ALGEBRAS


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Leibniz algebras were introduced in 1993 as a generalization of Lie algebras. In [3], it is proved that the Leibniz algebra having non-degenerate derivation is nilpotent. But the opposite is generally not true, i.e. there are nilpotent Leibniz algebras in which all derivations are nilpotent (and therefore degenerate). In [2], it is proved that the Leibniz algebra is nilpotent if and only if it has Leibniz derivations. Moreover, it is shown that every nilpotent Leibniz algebra of nilindex $s$ has a non-degenerate Leibniz derivation of order $\left[\frac{s}{2}\right] + 1$ .

In this paper, we present a strongly nilpotent Leibniz filiform algebra, for which all Leibniz derivation of order 3 are nilpotent. In addition, an example of a Leibniz algebra with non-nilpotent Leibniz-derivation of order 4 is given.

Definition 1.[1] An algebra $(L, \ [-,-])$ over a field $F$ is called *a Leibniz algebra*, if for any $x, y, z \in L$, the so-called Leibniz identity

$$[x, [y, z]] = [[x, y], z] - [[x, z], y]$$

holds.

For the given Leibniz algebra $L$ we consider the following central lower series:

$$L^{[1]} = L, \quad L^{[n+1]} = [L^{[n]}, L] \quad n \geq 1.$$

DEFINITION 2.[1] A Leibniz algebra $L$ is called *nilpotent*, if there exists $n \in N$ such that $L^{[n]} = 0$.

DEFINITION 3.[1] A linear transformation $d$ of the Leibniz algebra $L$ is called *derivation*, if for any $x, y \in L$ :

$$d([x, y]) = [d(x), y] + [x, d(y)].$$

DEFINITION 4. A linear transformation $P$ is called a Leibniz-derivation of order $n$ if the following is true,

$$P([[[x_1, x_2], x_3], \dots, x_n]) = [[[P(x_1), x_2], x_3], \dots, x_n] + [[[x_1, P(x_2)], x_3], \dots, x_n] + \\ + [[[x_1, x_2], P(x_3)], \dots, x_n] + \dots + [[[x_1, x_2], x_3], \dots, P(x_n)].$$

Consider the 9−dimensional nilpotent Leibniz algebra with the table of multiplication table:

$$L(1, \alpha_5, \alpha_6, \alpha_7, \alpha_8, \alpha_9, \theta) : \begin{cases} [e_1, e_1] = e_3, & \\ [e_i, e_1] = e_{i+1}, & 2 \le i \le 8, \\ [e_1, e_2] = e_4 + \sum_{t=5}^{8} \alpha_t e_t + \theta e_n, & \\ [e_j, e_2] = e_{j+2} + \sum_{t=j+3}^{n} \alpha_{t-j+2} e_t, & 2 \le j \le 7. \end{cases}$$

(the omitted products are zero).

It should be noted that the nilindex of this algebra is 9, i.e. $s = 9$.

**Proposition 1.** *Any Leibniz-derivation of order* 3 *of the algebra* $L(1, -2, 5, -14, 42,$-$132, \theta)$ *is nilpotent. Moreover, the matrix form of any Leibniz-derivation of order* 3 *has the following form:*

$$P = \begin{pmatrix} 0 & 0 & a_3 & a_4 & a_5 & a_6 & a_7 & a_8 & a_9 \\ 0 & 0 & a_3 & a_4 & a_5 & a_6 & b_7 & b_8 & b_9 \\ 0 & 0 & 0 & a_3 & a_4 & a_5 & c_7 & c_8 & c_9 \\ 0 & 0 & 0 & 0 & a_3 & a_4 & a_5 & a_6 & a_7 \\ 0 & 0 & 0 & 0 & 0 & a_3 & a_4 & a_5 & c_7 \\ 0 & 0 & 0 & 0 & 0 & 0 & a_3 & a_4 & a_5 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_3 & a_4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

**Proposition 2.** *There is a non-nilpotent Leibniz-derivation of order* 4 *of the algebra* $L(1, -2, 5, -14, 42, 132, \theta)$. *Moreover, the matrix form of any Leibniz-derivation of order* 4 *has the following form:*

$$P = \begin{pmatrix} a_1 & a_1 & a_3 & a_4 & a_5 & a_6 & a_7 & a_8 & a_9 \\ 0 & 2a_1 & a_3 & a_4 & a_5 & a_6 & b_7 & b_8 & b_9 \\ 0 & 0 & 3a_1 & a_3 + a_1 & a_4 + 2d_5 - 6a_1 - 2a_3 & a_5 + 2d_6 - 2a_4 & c_7 & c_8 & c_9 \\ 0 & 0 & 0 & 4a_1 & d_5 & d_6 & d_7 & d_8 & d_9 \\ 0 & 0 & 0 & 0 & 5a_1 & a_3 + 3a_1 & a_4 - 6a_1 & a_5 + 15a_1 & a_6 - 42a_1 \\ 0 & 0 & 0 & 0 & 0 & 6a_1 & a_3 + 4a_1 & a_4 - 12a_1 - 2a_3 + 2d_5 & a_5 + 2d_6 - 2a_4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 7a_1 & d_5 + 3a_1 & d_6 - 6a_1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 8a_1 & a_3 + 6a_1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 9a_1 \end{pmatrix}$$

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## EXISTENCE OF POINT OF THE ALGEBRA TO APPROXIMATE WITH THE EVOLUTION ALGEBRA

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In [1] a notion of evolution algebra is introduced. This evolution algebra is defined as follows. Let $(E, \cdot)$ be an algebra over a field $K$. If it admits a countable basis $e_1, e_2, \ldots, e_n, \ldots$, such that $e_i \cdot e_j = 0$, if $i \neq j$ and $e_i \cdot e_i = \sum_k a_{ik} e_k$, for any $i$, then it is called an *evolution algebra*. The matrix $(a_{ik})$ is known as matrix of structural constants.

In [2] and [3] a notion of approximation of evolution algebras is introduced. Here we introduce a notion of an approximation of arbitrary algebra with an evolution algebra.

Let $A$ be a $n$-dimensional algebra with basis $e_1, e_2, ..., e_n$. A multiplication on $A$ is defined by the multiplication of basis elements: $e_i e_j = \sum_k \gamma_{ij,k} e_k$.

For given matrix $(\gamma_{ij,k})$ we consider an evolution operator $F : x'_k = \sum_{i,j} \gamma_{ij,k} x_i x_j$. Jacobian of this operator is: $J_F(x) = (\sum_{i=1}^n (\gamma_{pi,k} + \gamma_{ip,k}) x_i)_{p,k=1}^n$. Since this operator is square we can define an evolution algebra with the matrix is $J_F(x)$ of structural constants. We denote $\beta_{pk}(x) = \sum_i (\gamma_{pi,k} + \gamma_{ip,k}) x_i$ and so we define an evolution algebra $\tilde{A}_x$ with basis $\tilde{e_1}, \tilde{e_2}, ..., \tilde{e_n}$ and so $\tilde{e_i}\tilde{e_j} = 0,\ i \neq j$; $\tilde{e_i}^2 = \sum_k \beta_{ik}(x) \tilde{e_k}$.

Thus

$$\beta_{pk}(x) = \sum_i (\gamma_{pi,k} + \gamma_{ip,k}) x_i \tag{1}$$

is a relation between the algebra $A$ and the evolution algebra $\tilde{A}_x$.

Therefore, $\tilde{A}_x$ is called an approximation of $A$ at $x \in A$.

Let $E$ be the given evolution algebra with matrix of structural constants $(a_{pk})$. We write the relation by formula (1):

$$a_{pk} = \sum_i (\gamma_{pi,k} + \gamma_{ip,k}) x_i. \tag{2}$$

Let $A$ be an algebra with matrix of structural constants $M_A = (\gamma_{pi,k})_{p,i,k=1}^n$, and $I_A = \{p \in \{1, 2, ..., n\} : det\Gamma_p \neq 0\}$, where $\Gamma_p = (\gamma_{pi,k} + \gamma_{ip,k})_{i,k=1}^n$, $a_p$ is a vector considered as $p$-th column of matrix $(a_{pk})$.

**Theorem.** *For a given non-trivial evolution algebra with* $\mathcal{M}_{EA} = \left(\beta^*_{pk}\right)$ *and an algebra* $A$ *with* $\mathcal{M}_A = (\gamma_{pi,k})$ *there is an element* $x \in A\backslash\{0\}$ *satisfying (2) if and only if*

$$\Gamma_p^{-1} \cdot a_p = \Gamma_q^{-1} \cdot a_q, \ \ for \ \ any \ \ p, q \in I_A$$

*and*

$$rank\Gamma_p = rank\left(\Gamma_p, a_p\right), \ \ for \ \ any \ \ p = 1, 2, ..., n\backslash I_A.$$

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## THE FUNCTOR OF IDEMPOTENT PROBABILITY MEASURES WITH COMPACT SUPPORT AND OPEN MAPS


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On the set $\mathbb{R}\bigcup\{-\infty\}$ let we define operations $\oplus$ and $\odot$ by the rules: $u \oplus v = \max\{u, v\}$ and $u \odot v = u + v$. Concerning these operations $-\infty$ is zero $\mathbf{0}$, and usual zero 0 is unit $\mathbf{1}$ on a set $\mathbb{R}\bigcup\{-\infty\}$. System $(\mathbb{R}\bigcup\{-\infty\}, \oplus, \odot, \mathbf{0}, \mathbf{1})$ forms the semi-field which can be designated by $\mathbb{R}_{\max}$.

Let $X$ be a Hausdorff compact space, $C(X)$ be the Banach algebra of all continuous functions defined on $X$, equipped with the usual pointwise algebraic operations and sup-norm. Introduce the next operations:

1) $\odot : \mathbb{R} \times C(X) \to C(X)$ by a rule $\odot(\lambda, \varphi) = \lambda \odot \varphi = \varphi + \lambda_X$, where $\varphi \in C(X)$ and $\lambda_X$ is constant function accepting everywhere on $X$ the value $\lambda \in \mathbb{R}$;

2) $\oplus : C(X) \times C(X) \to C(X)$ by a rule $\oplus(\varphi, \psi) = \varphi \oplus \psi = \max\{\varphi, \psi\}$, where $\varphi, \psi \in C(X)$.

**Definition 1.** A functional $\mu : C(X) \to \mathbb{R}$ is called an idempotent probability measure on $X$ if it satisfies the following properties:

(i) $\mu(\lambda_X) = \lambda$ for any $\lambda \in \mathbb{R}$;

(ii) $\mu(\lambda \odot \varphi) = \lambda \odot \mu(\varphi)$ for any $\lambda \in \mathbb{R}$ and $\varphi \in C(X)$;

(iii) $\mu(\varphi \oplus \psi) = \mu(\varphi) \oplus \mu(\psi)$ for any $\varphi$, $\psi \in C(X)$.

The number $\mu(\varphi)$ is called the Maslov's integral corresponding to $\mu$. The set of all idempotent probability measure on $X$ is denoted by $I(X)$. We have $I(X) \subset \mathbb{R}^{C(X)}$. Consider

$I(X)$ with induced from $\mathbb{R}^{C(X)}$ topology. The base of neighbourhoods of an idempotent probability measure $\mu \in I(X)$ concerning this topology consists of the sets of the look

$$\langle \mu;\ \varphi_1,\ ...,\ \varphi_n;\ \varepsilon\rangle = \{\nu \in I(X):\ |\nu(\varphi_i) - \mu(\varphi_i)| < \varepsilon,\ i = 1,\ ...,\ n\}$$

where $\varphi_i \in C(X)$, $i = 1,\ ...,\ n$, and $\varepsilon > 0$.

For any compact $X$ the space $I(X)$ is also a compact.

Let $f : X \to Y$ be a continuous map of compacts. Then the equality $I(f)(\mu)(\varphi) = \mu(\varphi \circ f)$, $\mu \in I(X)$, $\varphi \in C(Y)$, defines a continuous map $I(f) : I(X) \to I(Y)$.

For an idempotent probability measure $\mu \in I(X)$ one can define its support:

$$\text{supp}\mu = \bigcap \{F \subset X : F \text{ is a closed subset of } X \text{ such that } \mu \in I(F)\}.$$

Let $X$ is Tychonoff space, $\beta X$ its Stone-Čech compact extension. Let's put

$$I_\beta(X) = \{\mu \in I(\beta X) : \text{supp}\mu \subset X\}.$$

If $f : X \to Y$ is continuous map of Tychonoff spaces, $I(\beta f)(I_\beta(X)) \subset I_\beta(Y)$, where $\beta f : \beta X \to \beta Y$ map continuation $f$. Put

$$I_\beta(f) = I(\beta f) \mid I_\beta(X).$$

For a Tychonoff space elements of $I_\beta(X)$ are called as idempotent probability measures with finite support. The operation $I_\beta$ is the functor acting in the category $Tych$.

Recall that a continuous map $f\colon X \to Y$ is said to be an *open map* if the image of every open set $G \subset X$ is open in $Y$.

The following theorem is the main result of the article.

**Theorem 1.** *Let $f : X \to Y$ be a continuous map of Tychonoff spaces $X$ and $Y$. The map $I_\beta(f)\colon I_\beta(X) \to I_\beta(Y)$ is open if and only if $f$ is open.*

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## APPROXIMATION OF INCREASED ACCURACY OF THE PONTRYAGIN'S LOWER OPERATOR FOR ONE CLASS OF DIFFERENTIAL GAMES

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Let us consider the differential game

$$\dot{z} = f(z, u, v), \tag{1}$$

where $z \in \mathbb{R}^d, u \in U, v \in V, f : \mathbb{R}^d \times U \times V \longrightarrow \mathbb{R}^d$ is continuous function, $U$ and $V$ are convex compact subsets of $\mathbb{R}^p$ and $\mathbb{R}^q$, respectively. Along with system (1), we also fix the set $M$, $M \subset \mathbb{R}^d$, which is called the terminal set.

**Definition 1.** The operator $P_\varepsilon$ assigns to each set $A \subset \mathbb{R}^d$ the set $P_\varepsilon$ of all points $\xi \in \mathbb{R}^d$ such that there exists admissible control $u(\cdot) \in U[0, \varepsilon]$ of the pursuer for any admissible controls $v(\cdot) \in V[0, \varepsilon]$ of the evader the corresponding trajectory $z(t, u(\cdot), v(\cdot), \xi)$ of the system (1) with the beginning at the point $\xi \in \mathbb{R}^d$ г hits $A \subset \mathbb{R}^d$ at the time $\varepsilon$, i.e. $z(\varepsilon) \in A$. By means of operations of association and intersection we can write the operator $P_\varepsilon$ as follows

$$P_\varepsilon M = \bigcup_{u(\cdot)\in U[0,\varepsilon]} \bigcap_{v(\cdot)\in V[0,\varepsilon]} \{\xi \in \mathbb{R}^d : z(\varepsilon, u(\cdot), v(\cdot), \xi) \in M\}.$$

Let $\omega = 0 = \tau_0 < \tau_1 < ... < \tau_{n-1} < \tau_n = \tau$ be partition of the segment $[0, \tau]$ and $\delta_i = \tau_i - \tau_{i-1}$, $\mid \omega \mid= \tau$. We assume

$$P_\omega M = P_{\delta_1} P_{\delta_2} ... P_{\delta_n}.$$

**Definition 2.** $P_\tau M = \bigcup\limits_{|\omega|=\tau} P_\omega M.$

The operator $P_\tau$ is called the lower Pontryagin's operator of nonlinear differential games pursuit with fixed time [1-3].

In general the construction of approximate formulas of higher accuracy for the operator $P_\tau M$ is fraught with certain difficulties. Therefore, we further assume that $f(z, u, v) = \varphi(z) +$

$B(u,v)$, where $\varphi(z)$ is three times continuously differentiable function, $B(u,v)$ is continuous function and $B(u,Q)$ is convex set for any $u \in U$.

Consider the following operator

$$\Pi_\varepsilon M = \bigcup_{u\in U} \bigcap_{v\in V} \{z : z + [\varepsilon + \frac{1}{2}\varphi'_z(z)\varepsilon^2]\varphi(z) + [\varepsilon + \frac{1}{2}\varphi'_z(z)\varepsilon^2]B(u,v) \in M\}$$

**Theorem 1.** *The following equality holds*

$$P_\tau M = \bigcup_{\delta>0} \Pi_\tau(M \underline{*} \delta H),$$

*for open $M$, $M \subset \mathbb{R}^d$.*

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## ESTIMATE OF THE REMAINDER TERM IN CLT FOR THE NUMBER OF EMPTY CELLS AFTER ALLOCATION OF PARTICLES


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Let $s$ types of particles be allocated independently of each other and consistently in $N$ cells, the number of $l$-type particles be equal to $n_l$, each of them falls into the cell with the number $m$ with the probability

$$p_{lm} > 0, \quad m = 1, \dots, N, \quad p_{l1} + \dots + p_{lN} = 1, \quad l = 1, \dots, s.$$

Consider the following random variable $\mu_0(s)$ – the number of empty cells after the allocation of all $n_1, \dots, n_s$ particles. Evidently, each type of particles is allocated by the polynomial scheme, i.e. the random vector $\eta_i = (\eta_{l1}, \dots, \eta_{lN})$ can be set by the conditional distribution of the random vector $\xi_i = (\xi_{l1}, \dots, \xi_{lN})$, where we have

$$\pounds\left(\xi_{im}\right) = \Pi\left(n_i p_{im}\right), \quad m = 1, \dots, N, \quad l = 1, \dots, s$$

for independent random variables $\xi_{im}$.

Set $\xi_i^{(s)} = (\xi_{l1}, \dots, \xi_{lN})$, $\lambda_{lm} = n_l p_{lm}$, $\alpha_l = n_l/N$, $\lambda_m = \lambda_{1m} + \dots + \lambda_{sm}$,

$$A_n(s) = \sum_{m=1}^{N} \exp\left(-\lambda_m\right), \quad \gamma_l = \frac{1}{n_l}\sum_{m=1}^{N} \lambda_{lm} \exp\left(-\lambda_m\right),$$

$$\sigma_N^2(s) = \sum_{m=1}^{N} \exp(-\lambda_m) \left(1 - \exp(-\lambda_m) - \sum_{m=1}^{N} \alpha_l \gamma_l^2\right).$$

We suppose that for each $l = 1, \dots, s$

$$\max N p_{lm} \le C_0, \quad \ln \alpha_l \le \varepsilon N.$$

**Theorem.** *There exists $C(s)$ such that*

$$\Delta_N^{(s)}(y) = \left| P\left\{ \frac{\mu_0(s) - A_n(s)}{\sigma_N(s)} < y \right\} - \Phi(y) \right| \le C(s) \left[ \sigma_N^{-1}(s) + \sum_{m=1}^{N} \left(n_j\right)^{-1} \right] \Big].$$


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# THE NEW BOUNDARY VALUE PROBLEM FOR THE LOADED THIRD ORDER HYPERBOLIC TYPE EQUATION IN AN INFINITE THREE DIMENSIONAL DOMAIN


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Let $\Omega$ be three dimensional domain, bounded with the following surfaces:

$$\Gamma_0: \ \ 0 < x < 1, \ \ y = 0, \ \ -\infty < z < +\infty, \quad \Gamma_1: \ \ x + y = 0, \ \ 0 \le x \le \frac{1}{2}, \ \ -\infty < z < +\infty,$$

$$\Gamma_2: \ \ x - y = 1, \ \ \frac{1}{2} \le x \le 1, \ \ -\infty < z < +\infty.$$

We consider the following equation in infinite three dimensional domain $\Omega$

$$\frac{\partial}{\partial x}(U_{xx} - U_{yy} + U_{zz}) - \mu U(x, 0, z) = 0, \quad \mu = const < 0. \tag{1}$$

**BCG problem (The analog of the Darboux problem).** Find the function $U(x, y, z)$ with the following properties:

1) the function $U(x,y,z)$ is continuous till the bounder of the domain $\Omega$;

2) $U_x \in C(\Gamma_1)$, $U_y \in C(\Omega \bigcup \Gamma_0 \bigcup \Gamma_1)$

3) $U_{xxx}$, $U_{xzz}$, $U_{xyy} \in C(\Omega)$ and satisfy the equation (1) on the domain $\Omega$;

4) $U(x,y,z)$ satisfy the conditions:

$$\lim_{y\to -0} U(x,y,z) = F_1(x,z), \quad 0 \le x < 1, \;\; z \in (-\infty, +\infty),$$

$$U(x,y,z)_{|\Gamma_1} = F_2(x,z), \;\; \frac{\partial U(x,y,z)}{\partial n}_{|\Gamma_1} = F_3(x,z), \;\; 0 \le x \le \frac{1}{2}, \;\; z \in (-\infty, +\infty),$$

$$\lim_{|z|\to\infty} U = \lim_{|z|\to\infty} U_x = \lim_{|z|\to\infty} U_y = \lim_{|z|\to\infty} U_z = 0,$$

$n-$ is the internal normal, $F_j(x,z), (j = \overline{1,3})$ - are the given sufficiently smooth functions, where

$$\lim_{|z|\to\infty} F_j(x,z) = 0, \;\; (j = \overline{1,3}). \tag{2}$$

The main method of the study of the **BCG** problem is the Fourier transformation [1, 2]. Under the Fourier transformation at the determined constrains to the given functions the unique solvalility of the **BCG** problem is proved.

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## NUMERICAL SOLUTION OF PARABOLIC EQUATION WITH A DOUBLE NONLINEARITY WITH JUMPING


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In this work in the domain $Q = \{(t,x) : t > 0,\, x \in R_+\}$ is investigated the following problem

$$\frac{\partial u}{\partial t} = \frac{\partial}{\partial x}\left(u^{m-1}\left|\frac{\partial u^k}{\partial x}\right|^{p-2}\frac{\partial u}{\partial x}\right) - \gamma u^{q_1}\left|\frac{\partial u^m}{\partial x}\right|^{p_1}, \;\; x > 0, t > 0, \tag{1}$$

$$u(x,\,0) = 0, \;\; u(0,\,t) = u_0, \;\; u(t,\,l(t)) = 0, \tag{2}$$

where $m,\, k,\, p,\, \alpha, \beta, q_1,\, p_1$ are given numerical parameters. The equation (1) describes the processes of heat conductivity, diffusion, biological population and other different processes [1-5]. Equation (1) is degenerate, because of which, it in an area where $u = 0$ $\nabla u = 0$ may not have a solution in the classical sense. Therefore, it is necessary to study the weak solutions with properties

$$0 \le u, u^{m-1}\left|\frac{\partial u^k}{\partial x}\right|^{p-2}\frac{\partial u}{\partial x} \in C(Q), \tag{3}$$

and satisfy to the some integral identity in tense of distribution [1-3]. H. Zhan [1] applying the standard iteration method in the case $k = m$, a sufficient condition established to the existing of the singular self-similar solutions of the equation (1). Author gives a classification of these singular self- similar solutions. Various qualitative properties of the solution of the problem (1) and nonlinear phenomena for different particular value of the numerical parameters intensively studied by many authors (see [1-3] and references therein).

In this work using balance method [4] constructed the following approximately weak solution of the problem (1), (2)

$$\bar{u}(t,x) = (1 - \frac{x}{l(t)})_+^{\ \lambda}, \ \lambda = \frac{p - p_1}{m + k(p-2) - (p_1 + q_1)}, \ (n)_+ = \max(o, n)$$

The function $l(t)$ founded from solution of an nonlinear algebraic equation

$$l'(t) = {u_0}^{m+k(p-2)-1}(k\lambda)^{p-2}\lambda^2/l^{p-2}(t) + \lambda^{p_1+1}{u_0}^{p_1+q_1-1}/l^{p_1-1}(t), \ l(0) = 0$$

**Theorem.** *Let* $u(t, x)$ *be weak solution of the problem (1,2). Then for the weak solution the estimate* $u(t,x) \le A\bar{u}(t,x)$ *if* $1)p - \alpha > 0, \ m + k(p-2) - (\alpha+\beta) > 0,$ *if* $2)p - \alpha < 0, \ m + k(p-2) - (\alpha+\beta) < 0,$ *and* $u(0,x) \le u_0, \ x \in R_+$ *holds in* $Q$*. Where* $A$ *is constant, the functions* $\bar{u}(t,x)$, $l(t)$ *defined above.*

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# REGULARITY OF A VOLTERRA CUBIC STOCHASTIC OPERATOR

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A *cubic stochastic operator* (CSO), mapping the simplex

$$S^{m-1} = \{\mathbf{x} = (x_1, \dots, x_m) \in R^m : x_i \geq 0, \sum_{i=1}^{m} x_i = 1\}$$

into itself, is of the form

$$W : x'_k = \sum_{i,j,l=1}^{m} p_{ijl,k} x_i x_j, \quad (k = 1, \dots, m), \tag{1}$$

where

$$p_{ijl,k} \geq 0, \quad \sum_{k=1}^{m} p_{ijl,k} = 1, \quad (i, j, l, k = 1, \dots, m). \tag{2}$$

A Volterra CSO is defined by (1), (2) and by the assumption $p_{ijk,l} = 0$ if $l \notin \{i, j, k\}$.

The *trajectory* $\{\mathbf{x}^{(n)}\}$ for $\mathbf{x}^{(0)} \in S^{m-1}$ induced by the CSO (1) with parameters (2) is defined by $\mathbf{x}^{(n+1)} = W(\mathbf{x}^{(n)})$, where $n = 0, 1, 2, \dots$

A point $\mathbf{x}^* \in S^{m-1}$ is called a *fixed point* of a CSO $W$ if $W(\mathbf{x}^*) = \mathbf{x}^*$.

Let $DW(\mathbf{x}^*) = \Big((\partial W_i / \partial x_j)(\mathbf{x}^*)\Big)_{i,j=1}^{\infty}$ be the Jacobi matrix of $V$ at the point $\mathbf{x}^*$.

A fixed point $\mathbf{x}^*$ is called *hyperbolic* if its Jacobi matrix $DW(\mathbf{x}^*)$ has no eigenvalues 1 in absolute value. A hyperbolic fixed point $\mathbf{x}^*$ is called:(i) *attracting* (resp. *repelling*) if all the eigenvalues of the Jacobi matrix $DW(\mathbf{x}^*)$ are less (resp. greater) than 1 in absolute value; (ii) a *saddle* otherwise.

A CSO $W$ is called regular if the limit $\lim_{n\to\infty} W^n(\mathbf{x})$ exists for any $\mathbf{x} \in S^{m-1}$.

Let the set $\text{int}S^{m-1} = \{\mathbf{x} \in S^{m-1} : x_1 x_2 \cdots x_m > 0\}$ be the interior of $S^{m-1}$ and let $\Gamma_{i_1 \cdots i_j} = \text{conv}\{\mathbf{e}_{i_1} \cdots \mathbf{e}_{i_j}\}$.

We consider the following Volterra cubic stochastic operator defined on the two-dimensional simplex:

$$W : x'_1 = x_1\big(1 + x_3^2 - x_1 x_2\big), \quad x'_2 = x_2\big(1 + x_1^2 - x_2 x_3\big), \quad x'_3 = x_3\big(1 + x_2^2 - x_1 x_3\big). \tag{3}$$

**Theorem 1.** *For the Volterra CSO $W$ the following statements are true:*
*i) The vertexes $\mathbf{e}_1 = (1, 0, 0); \mathbf{e}_2 = (0, 1, 0); \mathbf{e}_3 = (0, 0, 1)$ are non-hyperbolic points and the center $\mathbf{c} = (1/3, 1/3, 1/3)$ of $S^2$ is an attracting fixed point of the operator $W$;*
*ii) If $\mathbf{x}^{(0)} \in \Gamma_{i,j}$ then $\lim_{n\to\infty} \mathbf{x}^{(n)} = \mathbf{e}_i$ for any $i \neq j \in \{1, 2, 3\}$;*
*iii) If $\mathbf{x}^{(0)} \in intS^2$ then $\lim_{n\to\infty} \mathbf{x}^{(n)} = \mathbf{c}$.*

**Corollary 2.** *The Volterra CSO $W$ is regular.*

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## HYPERSPACE OF THE WEAK Π-COMPLETE SPACES

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In the present paper under space we mean a topological $T_1$-space, under compact a Hausdorff compact space.

A collection $\omega$ of subsets of a set $X$ is said to be *star-countable* (respectively, *star-finite*) if each element of $\omega$ intersects at most a countable (respectively, finite) set of elements of $\omega$. A collection $\omega$ of subsets of a set $X$ *refines* a collection $\Omega$ of subsets of $X$ if for each element $A \in \omega$ there is an element $B \in \Omega$ such that $A \subset B$. They also say that $\omega$ is *a refinement* of $\Omega$. For a point $x \in X$ and a natural number $n$ they write $Kp(x, \omega) \leq n$, if not more than $n$ elements of $\omega$ contain $x$. We write $Kp\,\omega \leq n$ if $Kp(x, \omega) \leq n$ for every $x \in X$.

A finite sequence of subsets $M_0, \ldots, M_s$ of a set $X$ is a *chain* connecting sets $M_0$ and $M_s$, if $M_{i-1} \cap M_i \neq \varnothing$ for $i = 1, \ldots, s$. A collection $\omega$ of subsets of a set $X$ is said to be *connected* if for any pair of sets $M, M' \subset X$ there exists a chain in $\omega$ connecting the sets $M$ and $M'$. The maximal connected subcollections of $\omega$ are called *components* of $\omega$. A star-finite open cover of a spaces $X$ is said to be *a finite-component cover* if the number of elements of each component is finite.

For a collection $\omega = \{O_\alpha : \alpha \in A\}$ of subsets of a space $X$ we put $[\omega] = [\omega]_X = \{[O_\alpha]_X : \alpha \in A\}$. For a space $X$, its some subspace $W$ and a point $x \in X \setminus W$ they say that an open cover $\lambda$ of the space $W$ pricks out the point $x$ in $X$ if $x \notin \cup[\lambda]_X$. A space $X$ is said to be *weak* Π*-complete* if for every point $x \in \beta X \setminus X$ there exists an open-closed cover $\omega$ of $X$ which pricks out the point $x$ in $\beta X$. A closed subset of a weak Π-complete space is weak Π-complete.

Let $X$ be a space. By $\exp X$ we denote a set of all nonempty closed subsets of $X$. A family of sets of the view

$$O\langle U_1, \ldots, U_n\rangle = \{F \in \exp X : \ F \subset \bigcup_{i=1}^{n} U_n, F \cap U_1 \neq \varnothing, \ldots, F \cap U_n \neq \varnothing\}$$

forms a base of a topology on $\exp X$, where $U_1, \ldots, U_n$ are open nonempty sets in $X$. This topology is called *the Vietoris topology.* A space $\exp X$ equipped with the Vietoris topology is called *hyperspace* of $X$. For a compact $X$ its hyperspace $\exp X$ is also a compact.

Let $f\colon X \to Y$ be a continuous map of compacts, $F \in \exp X$. We put

$$(\exp f)(F) = f(F).$$

For a Tychonoff space $X$ we put

$$\exp_\beta X = \{F \in \exp \beta X : F \subset X\}.$$

It is clear, that $\exp_\beta X \subset \exp X$. Consider the set $\exp_\beta X$ as a subspace of the space $\exp X$. For a Tychonoff spaces $X$ the space $\exp_\beta X$ is also a Tychonoff space with respect to the induced topology.

For a continuous map $f\colon X \to Y$ of Tychonoff spaces we put

$$\exp_\beta f = (\exp \beta f)|_{\exp_\beta X},$$

where $\beta f\colon \beta X \to \beta Y$ is the Stone-Cěch compactification of $f$ (it is unique).

It is well known that for a Tychonoff space $X$ the set $\exp_\beta X$ is everywhere dense in $\exp \beta X$, i. e. $\exp \beta X$ is a compactification of the space $\exp_\beta X$.

Recall a notion of the perfect compactification. For a topological space $X$ and its subset $A$ a set $Fr_X A = [A]_X \cap [X \setminus A]_X = [A]_X \setminus Int_X A$ is called a boundary of $A$. We confirm that $\exp \beta X$ is a perfect compactification of $\exp_\beta X$.

**Theorem 1.** *For a Tychonoff space $X$ the space $\exp \beta X$ is a perfect compactification of the space $\exp_\beta X$.*

**Theorem 2.** *For a Tychonoff space $X$ its hyperspace $\exp_\beta X$ is weak $\Pi$-complete iff $X$ is weak $\Pi$-complete.*

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# THE SECOND COHOMOLOGY GROUP OF NONCOMMUTATIVE JORDAN ALGEBRA OF LEVEL ONE

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Degenerations of algebras is an interesting subject that was studied in various papers. One of important problem in this direction is the description of so-called rigid algebras. These algebras are of big interest, since the closures of their orbits under the action of generalized linear group form irreducible components of a variety under consideration (with respect to the Zariski topology). The method of degenerations is one of the tool of finding the rigid algebras. The degenerations of algebras can be represented by "arrow". Since there is no algebra that degenerates to a rigid algebra it can be easily concluded that the rigid algebras are located in the highest vertices of this representation scheme.

It is known that any $n$-dimensional algebra $\mathfrak{J}$ over a field $F$ is regarded as an element of $\lambda$ of affine variety. $Hom(V \bigotimes V, V)$ via the bilinear mapping $\lambda : V \bigotimes V \to V$ on underlying vector space $V$ of $L$.

Since the space $Hom(V \bigotimes V, V)$ forms an $n^3$-dimensional affine space $B(V)$ over $F$, we will consider the Zariski topology on this space and the linear reductive group $GL_n(F)$ acting on the space as follows:

$$(g * \lambda)(x, y) = g(\lambda(g^{-1}(x), g^{-1}(y))).$$

The orbits (Orb(-)) under this action are the isomorphism classes of algebras. It is clear that if a subvariety of $Hom(V \bigotimes V, V)$ is specified by an identity or identities like commutativity, skew-symmetricity, nilpotency, etc. then it is invariant under the action $*$. The closures of orbits under this action play a crucial role in the description of irreducible components of a variety of algebras.

**Definition 1.** An algebra $\lambda$ is said to degenerate an algebra $\mu$, if $Orb(\mu)$ lies in the Zariski closure of $Orb(\lambda)$. We denote this by $\lambda \to \mu$.

The degeneration $\lambda \to \mu$ is called a *direct degeneration* if there is no chain of non-trivial degeneration of the form: $\lambda \to \nu \to \mu$.

**Definition 2.** A level of an algebra $\lambda$ is the maximum length of chain of direct degenerations, which, of course ends with the algebra $\mathfrak{a}$(the algebra with zero multiplication). We denote the level of an algebra $\lambda$ by $lev_n(\lambda)$.
Consider $n$-dimensional noncommutative Jordan algebra $\nu_n(\alpha)$ with following multiplication table:

$$\nu_n(\alpha) : e_1e_1 = e_1, \quad e_1e_i = \alpha e_i, \quad e_ie_1 = (1 - \alpha)e_i, \qquad 2 \le i \le n.$$

In [1] it is shown that the algebra $\nu_n(\alpha)$ is algebra of level one.

**Definition 3.** An algebra $\mathfrak{J}$ is called *noncommutative Jordan algebra* if

$$(xy)x = x(yx),$$

$$(x^2y)x = x^2(yx).$$

Easy to show that the algebra $\nu_n(\alpha)$ is noncommutative Jordan algebra.

Let $\mathfrak{J}$ be a noncommutative Jordan algebra and $Z^2(\mathfrak{J},\mathfrak{J})$ be the space of the bilinear maps $h : \mathfrak{J} \times \mathfrak{J} \to \mathfrak{J}$ such that

$$h(xy, x) + h(x, y)x = xh(y, x) + h(x, yx)$$

$$(h(a, a)b)a + h(a^2, b)a + h(a^2b, a) = a^2h(b, a) + h(a^2, ba) + h(a, a)(ba).$$

If $\mu : \mathfrak{J} \to \mathfrak{J}$ is a linear mapping and $B^2(\mathfrak{J},\mathfrak{J})$ be the space of the bilinear maps $d\mu : \mathfrak{J} \to \mathfrak{J}$ given by

$$d\mu(a, b) = \mu(a)b + a\mu(b) - \mu(ab)$$

lies in $Z^2(\mathfrak{J},\mathfrak{J})$. The quotient space of $Z^2(\mathfrak{J},\mathfrak{J})$ by the space $B^2(\mathfrak{J},\mathfrak{J})$ is called the second cohomology group of $\mathfrak{J}$ and denoted by $H^2(\mathfrak{J},\mathfrak{J})$. Usually the space $Z^2(\mathfrak{J},\mathfrak{J})$ is called 2-cocycle and $B^2(\mathfrak{J},\mathfrak{J})$ is called 2-coboundary [2].

**Definition 4.** A noncommutative Jordan algebra $\mathfrak{J}$ is called cohomologically rigid if

$$H^2(\mathfrak{J},\mathfrak{J}) = 0.$$

It is well known that $\dim B^2(\mathfrak{J},\mathfrak{J}) = n^2 - \dim Der(\mathfrak{J})$, where $Der(\mathfrak{J})$ is all derivations of algebra $\mathfrak{J}$. Since $\dim(\nu_n(\alpha)) = n^2 - n$ we have $\dim B^2(\nu_n(\alpha), \nu_n(\alpha)) = n$.

**Lemma 5.** $\dim Z^2(\nu_3(\alpha), \nu_3(\alpha)) = 9$.

**Theorem 6.** $\dim H^2(\nu_3(\alpha), \nu_3(\alpha)) = 6$.

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## APPROACHES TO THE EVALUATION OF THE STATE OF A POORLY FORMALIZABLE PROCESS BASED ON A FUZZY INTEGRAL


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The main purpose of this work is to develop an algorithm for selecting breeding varieties of cotton with the best biological and technological indicators in terms of a vaguely specified initial information. And under such initial conditions, it is required to choose the most acceptable alternative: a variety for given sowing conditions, cultivation (agrotechnological regimes, components of fertilization dose, irrigation, boundary conditions for these varieties and soil types). The choice of alternatives can be made using a fuzzy integral. The experiment was carried out for the task of choosing from four breeding varieties: C-4727, Tashkent 1, 159-F, 108-F cotton ($X = \{x_1, x_2, ..., x_4\}$) best in the following characteristics ($P = \{p_1, p_2, ..., p_4\}$): yield, fiber length, fiber strength, seed oil content [2].

The importance of each feature is given and expressed through fuzzy densities.

$$g_1 = 0,66, \quad \text{g}_2\text{=0,89}, \ \text{g}_3 = 0,96, \quad g_4 = 0,93$$
$$h_1 = 0,19, \quad h_2\text{=0,21}, \ h_3 = 0,22, \quad h_4 = 0,24$$
$$g_\lambda(x_1,\, x_2,\, x_3,\, x_4) = 1.$$

$$g_1g_2g_3g_4\lambda^3 + (g_1g_2g_3 + g_1g_2g_4 + g_1g_3g_4 + g_2g_3g_4)\lambda^2 +$$
$$(g_1g_2 + g_1g_3 + g_1g_4 + g_2g_3 + g_2g_4 + g_3g_4)\lambda + g_1 + g_2 + g_3 + g_4 = 1.$$

$$0,524\lambda^3 + 2,49\lambda^2 + 4,409\lambda + 2,44 = 0.$$
$$\lambda^3 + 4,75\lambda^2 + 8,41\lambda + 4,66 = 0. \ \lambda = -0,96.$$
$$h_1 = 0,19, \quad h_2\text{=0,21}, \ h_3 = 0,22, \quad h_4 = 0,24.$$
$$\int h \circ g = \bigcup_{i=1}^{4} (h(x_i) \wedge g(E_i)) = \max(0,19;\ 0,21;\ 0,22;\ 0,24) = 0,24 \ x_4 = 0,24$$

Thus, the results of ranking all breeding varieties showed that grade 108-F is the best among the proposed breeding varieties of cotton, since the resulting value of the degree of belonging of this variety to a fuzzy set is greatest (0.24).

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## ON LATERALLY SIMPLE $L^0(\nabla)$-MODULES


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Let $\nabla$ be a complete Boolean algebra, let $Q = Q(\nabla)$ be a Stone compact corresponding to $\nabla$ and let $C_\infty(Q)$ be an algebra of all continuous functions $f : Q \to [-\infty, +\infty]$, taking the values $\pm\infty$ only on nowhere dense sets in $Q$. Let $X$ be an $C_\infty(Q)$-module with algebraic operations $x+y$ and $\alpha x$, $x, y \in X$, $\alpha \in C_\infty(Q)$. Since the Boolean algebra $\nabla$ and the Boolean algebra of all clopen subsets in $Q(\nabla)$ are isomorphic [2], then we denote the algebra $C_\infty(Q)$ by $L^0(\nabla)$.

We say that a regular $L^0(\nabla)$-module $X$ is laterally complete ($l$-complete), if for any set $\{x_i\}_{i\in I} \subset X$ and for any partition $\{e_i\}_{i\in I}$ of unity of the Boolean algebra $\nabla$ there exists $x \in X$ such that $e_ix = e_ix_i$ for all $i \in I$. In this case, the element $x$ is called mixing of the set $\{x_i\}_{i\in I}$ with respect to the partition of unity $\{e_i\}_{i\in I}$ and denote by $\underset{i\in I}{\text{mix}}(e_ix_i)$.

In [1] the following description of finite dimensional laterally complete $L^0(\nabla)$-modules is given

**Theorem 1.** *If $X$ is a finite-dimensional $L^0(\nabla)$-module, then there exist an uniquely defined finite partition $e_i \neq 0$, $i = 1, \ldots, k$, of unity in the Boolean algebra $\nabla$ and a finite set of positive integers $n_1 < \ldots < n_k$, such that the $L^0(\nabla)$-module $X$ is isomorphic to the $L^0(\nabla)$-module $\prod\limits_{i=1}^{k} (L^0(\nabla))_{e_i}^{n_i}$.*

Let $X$ be a faithful $l$-complete $L^0(\nabla)$-module. A submodule $Y$ in $X$ we call an $l$-submodule, if $\mathrm{mix}(Y) = Y$ and $s(Y) = \mathbf{1}$.

**Proposition 2.** *If $X = L^0(\nabla)$ and $Y$ is $l$-submodule in $X$, then $Y = X$.*

$l$-Submodule $Y$ in $X$ is called improper $l$-submodule if $Y = X$. All other $l$-submodules are called proper $l$-submodule.

$L^0(\nabla)$-module $X$ is called laterally simple ($l$-simple), if there is no proper $l$-submodules in $X$. It is not difficult to show, that $X = L^0(\nabla)$ is $l$-simple $L^0(\nabla)$-module.

**Proposition 3.** *If $X$ is finitely dimensional $l$-simple faithful $l$-complete $L^0(\nabla)$-module, then $X \approx L^0(\nabla)$.*

Propositions 2 and 3 imply the following

**Theorem 4.** *Faithful $l$-complete finitely dimensional $L^0(\nabla)$-module $X$ is $l$-simple iff $X \approx L^0(\nabla)$.*

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# ON THE NEGATIVITY OF HAUSDORFF ALGORITHMIC REPRESENTATIONS OF TRANSLATIONAL COMPLETE ALGEBRAS


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All unknown concepts can be found in the works of Yu.L.Ershov and S.S.Goncharov [1], A.I.Maltsev [2], N.H.Kasymov [3].

As usual, a total function acting from a set of natural numbers $\omega$ into $\omega$ is called computable if there exists an algorithm computing it ([4]). A subset $\alpha \subseteq \omega$ is called computable (positive, negative) if its characteristic function is computable ($\alpha$ is the range of a suitable computable function, $\alpha$ is a complement of the positive set).

Let $\langle A; \Sigma \rangle$ be a universal algebra of effective signature $\Sigma$. The mapping of $\nu$ from the set of natural numbers $\omega$ on $A$ is called algorithmic representation of the algebra $A$ if there exists a computable family $F$ of computable functions representing of the $\Sigma$ -operations of algebra $A$ in the representation $\nu$, i.e. for every $\Sigma$ -operation $\sigma \in \Sigma$ algebra $A$ there is a computable function $f \in F$ such that $\sigma\nu\overline{x} = \nu f \overline{x}$.

The algorithmic representation $\nu$ of algebra $A$ is called computable (positive, negative), if its kernel (i.e., the equivalence relation $Ker\,\nu = \{\langle x, y\rangle | \nu x = \nu y\}$) is such that.

Let $\eta$ be an equivalence on $\omega$. Subset $\alpha \subseteq \omega$ is called $\eta$ -closed if $x \in \alpha \wedge x = y \pmod{\eta} \rightarrow y \in \alpha$, i.e. if a $\alpha$ is the union of suitable $\eta$-classes.

The algorithmic representation of the $\nu$ algebra $A$ is called separable (computably separable), if for every pair of numbers, different in modulus $Ker\,\nu$, there is a $Ker\,\nu$-closed a positive (computable) set containing exactly one of these numbers.

Unary thermal operation with fixed elements of algebra in the quality of parameters is called translation (A.I. Maltsev [2]).

**Definition 1.** Universal algebra is called translational complete if every pair of its different elements is translated into any other pair of different elements by suitable translation.

Obviously, any translational complete algebra is simple. But the converse is not true.

The class of translational complete algebras is very rich. For example, any the division ring is translational complete ([5]).

It is known that every computable separable representation of any congruence-simple universal algebra is negative ([3]).

For the case of separable representation, the question is still open ([6]). A partial (positive) decision is announced as follows

**Theorem 1.** *Any Hausdorff separable algorithmic representation of any translational complete algebra is negative.*

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# A DESCRIPTION OF THE ORTHOGONAL GROUP OF THE TWO-DIMENSIONAL BILINEAR-METRIC SPACE WITH THE FORM $x_1y_1 + px_2y_2$ OVER THE FIELD OF RATIONAL NUMBERS


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Let $K$ be a field of rational numbers, $K^2$ be the 2-dimensional vector space over $K$ and $p$ be a prime number. Consider the following bilinear form $< x, y >= x_1y_1 + px_2y_2$ on $K^2$, where $x = (x_1, x_2), y = (y_1, y_2) \in K^2$. A mapping $A : K^2 \to K^2$ will be called orthogonal (form-preserving) if $< A(x), A(y) >=< x, y >$ for all $x, y \in K^2$. Denote by $O(2, K)$ the set of all orthogonal transformations of $K^2$. It is easy to see that every element of $O(2, K)$ is a linear transformation of $K^2$. Let $A \in O(2, K)$. Then $det(A) = 1$ or $det(A) = -1$. We denote by $SO(2, K)$ the set $\{A \in O(2, K)|det(A) = 1\}$. Put $U = \|u_{ij}\|_{i,j=1,2}$, where $u_{11} = 1, u_{12} = u_{21} = 0, u_{22} = -1$. Then $U \in O(2, K)$. It is easy to see that $O(2, K) = SO(2, K) \cup \{HU \mid H \in SO(2, K)\}$ holds, where $HU$ is the multiplication of matrices $H$ and $U$.

Let $x = (x_1, x_2) \in K^2$. Denote by $M(x_1, x_2)$ the matrix of the form $\begin{pmatrix} x_1 & -px_2 \\ x_2 & x_1 \end{pmatrix}$. Let $SM(K)$ be the set of all matrices $M(x_1, x_2)$ such that $det(M(x_1, x_2)) = x_1^2 + px_2^2 = 1$.

**Theorem 1.** *The equality $SO(2, K) = SM(K)$ holds.*

**Theorem 2.** (1) *There does not exist an element $x = (x_1, x_2) \in K^2$ such that $x_1 = 0$ and $M(x_1, x_2) \in SM(K)$. There exist only two elements $(x_1, x_2) \in K^2$ such that $x_2 = 0$ and $M(x_1, x_2) \in SM(K)$. They are $(1, 0)$ and $(-1, 0)$.*

(2) *Assume that $x = (x_1, x_2) \in K^2$ such that $x_2 \neq 0$ and $M(x_1, x_2) \in SM(K)$. Then the following equalities*

$$x_1 = \frac{p - k^2}{p + k^2}, x_2 = -\frac{2k}{p + k^2}, \tag{5}$$

*hold for some $k \in K$, where $k \neq 0$. Conversely, assume that $k$ is an arbitrary nonzero element of $K$ and $x = (x_1, x_2) \in K^2$ such that $x_1$ and $x_2$ have the forms Eq.(1). Then $M(x_1, x_2) \in SM(K)$.*

Theorem 2 implies a description of the group $O(2, K)$.

Results obtained above will be useful in the invariant theory of systems of points, curves and vector fields in $K^2$, in computer geometry and computational geometry.

# ON LEIBNIZ SUPERALGEBRAS WHICH EVEN PART IS SEMI-SIMPLE LIE ALGEBRAS

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During many years the theory of Lie superalgebras has been actively studied by many mathematicians and physicists. Many works have been devoted to the study of this topic, but unfortunately most of them do not deal with nilpotent Lie superalgebras. In works [2], [3] the problem of the description of some classes of nilpotent Lie superalgebras have been studied.

Our main focus in this work is to describe Leibniz superalgebras which even part is isomorphic to the three-dimensional simple Lie algebra $sl_2$. Note that odd part of the Leibniz superalgebra can be considered as a Leibniz bi-module of the even part. If the multiplication of the odd part is zero, then Leibniz superalgebra is isomorpic to Leibniz algebra, i.e., superalgebra with trivial odd part.

**Definition.** A $\mathbb{Z}_2$-graded vector space $L = L_0 \oplus L_1$ is called a Leibniz superalgebra if it is equipped with a product $[-,-]$ which satisfies the following conditions:

$$\big[x,[y,z]\big] = \big[[x,y],z\big] - (-1)^{\alpha\beta}\big[[x,z],y\big] - \text{Leibniz superidentity}$$

for all $x \in L, y \in L_\alpha, z \in L_\beta$.

The vector spaces $L_0$ and $L_1$ are said to be the even and odd parts of the superalgebra $L$, respectively. It is obvious that $L_0$ is a Leibniz algebra and $L_1$ is a representation of $L_1$.

**Theorem.** [1] *Let $L$ be a finite-dimensional Leibniz algebra, and let $M$ be a finite-dimensional irreducible $L$-bimodule. Then either $[L,M] = 0$ or $[x,m] = -[m,x]$ for all $x \in L$ and $m \in M$.*

**Lemma 1.** *Let $L = L_0 \oplus L_1$ be a Leibniz superalgebra, such that $L_0$ is a semi-simple Lie algebra. Then $[x,y] = [y,x]$ for any $x,y \in L_1$.*

**Lemma 2.** *Let $L = L_0 \oplus L_1$ be a Leibniz superalgebra, such that $L_0$ is a semi-simple Lie algebra and $[L_0,L_1] = 0$, then $[L_1,L_1] = 0$.*

In case of $[L_1, sl_2] = 0$, from Lemma 2, we derive that $[L_1,L_1] = 0$ and L is isomorphic to the Leibniz algebra with the following muliplication [5]:

$$\begin{cases} [e,h] = 2e, & [h,f] = 2f, & [e,f] = h, \\ [h,e] = -2e, & [f,h] = -2f, & [f,e] = -h, \\ [x_i,h] = (n-2i)x_i, & [x_i,f] = x_{i+1}, & [x_i,e] = -i(n-i+1)x_{i-1}. \end{cases}$$

**Proposition 1.** *Let $L = sl_2 \oplus L_1$ be a Leibniz superalgebra, such that $L_1$ is an irreducible bimodule. Let $dim L_1 = 2$ and $[x,y] = -[y,x]$ for all $x \in sl_2$, $y \in L_1$, then $L$*

*is isomorphic one of the following two Leibniz superalgebras:*

$$S_1 : \begin{cases} [e,h] = 2e, & [h,f] = 2f, & [e,f] = h, \\ [h,e] = -2e, & [f,h] = -2f, & [f,e] = -h, \\ [x_0,h] = x_0, & [h,x_0] = -x_0, & [x_1,h] = -x_1, \\ [h,x_1] = x_1, & [x_0,f] = x_1, & [f,x_0] = -x_1, \\ [x_1,e] = -x_0, & [e,x_1] = x_0. & \end{cases}$$

$$S_2 : \begin{cases} [e,h] = 2e, & [h,f] = 2f, & [e,f] = h, \\ [h,e] = -2e, & [f,h] = -2f, & [f,e] = -h, \\ [x_0,h] = x_0, & [x_1,h] = -x_1, & [h,x_0] = -x_0, \\ [h,x_1] = x_1, & [x_0,f] = x_1, & [f,x_0] = -x_1, \\ [x_1,e] = -x_0, & [e,x_1] = x_0, & [x_0,x_0] = 2e, \\ [x_1,x_1] = 2f, & [x_0,x_1] = h, & [x_1,x_0] = h. \end{cases}$$

**Proposition 2.** *Let $L = sl_2 \oplus L_1$ be a Leibniz superalgebra, such that $L_1$ is an irreducible bimodule. Let $dimL_1 \geq 3$ and $[x,y] = -[y,x]$ for all $x \in sl_2$, $y \in L_1$, then $[L_1, L_1] = 0$.*

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## CONVEX HULL GENERATED BY A POISSON POINT PROCESS


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Let $\Pi_n$ be a nonhomogeneous Poisson point process (nppp) with intensity

$$\lambda_n(A)=\begin{cases}\dfrac{1}{2\pi L(b_n)\sqrt{b_n}}\displaystyle\int\int\limits_A \frac{\partial}{\partial y}\left[\left(y-\frac{x^2}{2b_n}\right)^{\beta}L\left(\frac{b_n}{y-\frac{x^2}{2b_n}}\right)\right]dxdy, \text{ если } A\subset R_n\\ 0, \text{ если } A\not\subset R\end{cases}$$

where $L(x)$ is slowly changing function in the sense of Karamata and $R_n = \left\{(x,y): y\geq \frac{x^2}{2b_n}\right\}\subset R^2$ .

Recall that when we call such a point a vertex process $W_n(a)=(X_n(a),Y_n(a))$ and $a\in R$ let's call such a point $(X_k,Y_k)$ implementation of a (nppp) $\Pi_n$ , for which $X_k-aY_k$ takes the minimum value.

Put,$R_n(a)=X_n(a)-ab_n$ ,$S_n(a)=Y_n(a)-\frac{X_n^2(a)}{2b_n}+\frac{R_n^2(a)}{2b_n}$, $T_n(a)=(R_n(a),S_n(a))$ . In articles [1]-[2] probabilistic behavior, stationarity, and properties of strong mixing of the process were investigated $T_n(a)$ for any constant $a\in R$ .

Let $W_0=(X_0,Y_0), W_1=(X_1,Y_1),....., W_k=(X_k,Y_k)$ a sequence of vertices of a convex hull,$\tau_n(a)=\sup\left\{k:\frac{Y_k-Y_{k-1}}{X_k-x_{k-1}}\leq a\right\}$ . If we denote by $\xi_n^*$ Џ the area of the area bounded by lines $y-Y_{\tau_n(a)-1}=\frac{Y_{\tau_n(a)}-Y_{\tau_n(a)-1}}{X_{\tau_n(a)}-X_{\tau_n(a)-1}}(x-X_{\tau_n(a)-1})$ , $y-Y_{\tau_n(a)}=a(x-X_{\tau_n(a)})$ and a parabola $y=\frac{x^2}{2b_n}$. We put $A_n(a)=\sum\limits_{k\leq\tau(a)}\xi_k+\xi_n^*$ and denote by $l_n(a)$ Џ the length of the perimeter of the random broken connecting vertex $W_1,W_2,....,W_{\tau_n(a)}$, $t=(r,s)$,

$$M^k(t;R^2)=\frac{1}{2\pi\sqrt{b_n}L(b_n)}\int\limits_0^{\sqrt{2b_ns_n}}(u-r)^k\frac{\partial}{\partial s}$$

$$\left\{\left(s-\frac{u^2}{2b_n}\right)^{\beta}L\left(\frac{b_n}{s-\frac{u^2}{2b_n}}\right)\right\}du=\frac{1}{2\pi\sqrt{b_n}L(b_n)}\int\limits_0^{\sqrt{2b_ns_n}-r}u^k\frac{\partial}{\partial s}$$

$$\left\{\left(s-\frac{(u+r)^2}{2b_n}\right)^{\beta}L\left(\frac{b_n}{s-\frac{(u+r)^2}{2b_n}}\right)\right\}du.$$

The main result of this article is the following

**Theorem.** *Processes*

$$A_n(a)-\frac{1}{2}\int\limits_0^a(\sqrt{2b_ns(b)}-r(b))^2db,\quad l_n(a)-\int\limits_0^a\sqrt{1+b^2}M^{(2)}(T(b);R^2)db$$

*and*

$$l_n^2(a)-\int\limits_0^a\left\{(1+b^2)M^{(3)}(T(b);R^2)+2\sqrt{1+b^2}L(b)M^{(2)}(T(b);R^2)\right\}db$$

*form a martingale regarding $\sigma$-algebra*

$$F_a=\sigma\left\{T(c):0\leq c\leq a\right\}.$$

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## MODELING OF THE MULTIDIMENSIONAL PROBLEM OF NONLINEAR HEAT CONDUCTIVITY IN NON-DIVERGENTE CASE

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In the domain $D = \Omega \times (0,T)$, $\Omega \subset R^2$, $\Omega = \{a_\alpha < x_\alpha < b_\alpha, \quad \alpha = 1,2\}$ will be considered the following equation

$$Lu \equiv -u_t + P(u)\nabla\left(|x|^m K(u)\nabla u\right) + \varepsilon\,\gamma(t,x)Q(u) = 0, \quad x \in \Omega, \tag{1}$$

with initial

$$u(0,x) = u_0\left(x\right) \geq 0, \;\; a_\alpha \leq x_\alpha \leq b_\alpha, \quad \alpha = 1,2, \quad x \in \Omega, \tag{2}$$

and boundary conditions

$$u|_G = \mu(x,t), \quad t \in (0,T), \quad G - \text{boundary in } \Omega. \tag{3}$$

Here $P(u) = u^p$, $0 < p < 1$, $K(u) = |\nabla u|^{n-1}$, $\quad Q(u) = u^\beta$, $m \neq 0$, $0 < \gamma(t,x) \in C((0,T) \times [a_\alpha, b_\alpha]\ )$, $\varepsilon = \pm 1$.

Equation (1) has a non-divergent form, and is degenerate, it is in an area where $v(t,x) = 0$ it may not have a solution in the classical sense. Therefore, it is necessary to study those generalized solutions possessing the properties $0 \leq v(x,t), |x|^m \left|\nabla v^k\right|^{n-1} \nabla v \in C(Q)$, where, $Q = \left\{(t,x) : t > 0, x \in R^2\right\}$.

When $\varepsilon = -1$, $\gamma(t) = const$, an estimate of the front is

$$|x_f(t)| \leq \left[\frac{n+1-m}{n+1}\left(\frac{a}{b}\right)^{\frac{n}{n+1}} \cdot \tau_1(t)^{\frac{1}{n+1}}\right]^{\frac{n+1}{n+1-m}} = S(n,m) \cdot \tau_1^{\frac{1}{n+1-m}},$$

here $S(n,m) = \left[\frac{n+1-m}{n+1}\right]^{\frac{n+1}{n+1-m}} \left(\frac{a}{b}\right)^{\frac{n}{n+1-m}}$.

The problem is solved numerically on a computer and numerical experiments are performed in various values of the parameters. For numerical solution of the problem were applied the methods of alternating directions, the method of iterations. Using numerical experiments obtained important results of the problem in the two-dimensional case.

# ON $Z$-SETS OF THE SPACE OF IDEMPOTENT PROBABILITY MEASURES

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We will consider the set $\mathbb{R}_{\max} = \mathbb{R} \bigcup \{-\infty\}$ with two algebraic operations: addition $\oplus$ and multiplication $\odot$ determined as follows $u \oplus v = \max\{u, v\}$ and $u \odot v = u + v$ where $\mathbb{R}$ a set of real numbers.

Let $X$ be a Hausdorff compact space ($\equiv$ a compact), $C(X)$ be the algebra of continuous functions on $X$ with the usual algebraic operations. For $C(X)$ operations $\oplus$ and $\odot$ we define as follows: $\varphi \oplus \psi = \max\{\varphi, \psi\}$ and $\varphi \odot \psi = \varphi + \psi$, $\varphi$, $\psi \in C(X)$. Recall that functional $\mu : C(X) \to \mathbb{R}$ is called [1] idempotent probability measure on $X$, if it has the following properties: (1) $\mu(\lambda_X) = \lambda$ for all $\lambda \in \mathbb{R}$, where $\lambda_X$ is a constant function; (2) $\mu(\lambda \odot \varphi) = \lambda \odot \mu(\varphi)$ for all $\lambda \in \mathbb{R}$ and $\varphi \in C(X)$; (3) $\mu(\varphi \oplus \psi) = \mu(\varphi) \oplus \mu(\psi)$ for all $\varphi$, $\psi \in C(X)$;

For a compact set $X$ we denote by $I(X)$ the set of all idempotent probability measures on $X$. Consider $I(X)$ as a subspace of $\mathbb{R}^{C(X)}$. For given compacts $X$, $Y$ and a continuous mapping $f : X \to Y$ we can verify that the natural map $I(f) : I(X) \to I(Y)$, defined by the formula $I(f)(\mu)(\psi) = \mu(\psi \circ f)$, is continuous. Moreover, the construction $I$ is a normal functor. Therefore, for an arbitrary idempotent probability measure $\mu \in I(X)$ one can define the concept of support: $\text{supp}\mu = \bigcap \left\{A \subset X : \overline{A} = A,\ \mu \in I(A)\right\}$. For a positive integer $n$, we define the following set $I_n(X) = \{\mu \in I(X) :\ |\text{supp}\,\mu| \le n\}$. Set $I_\omega(X) = \bigcup\limits_{n=1}^{\infty} I_n(X)$. The set $I_\omega(X)$ is everywhere dense [1] in $I(X)$. Idempotent probability measure $\mu \in I_\omega(X)$ is called an idempotent probability measure with finite support. Note that if $\mu$ is an idempotent probability measure with finite support $\text{supp}\,\mu = \{x_1, x_2, ..., x_k\}$, then $\mu$ can be represented as $\mu = \lambda_1 \odot \delta_{x_1} \oplus \lambda_2 \odot \delta_{x_2} \oplus ... \oplus \lambda_k \odot \delta_{x_k}$ the only way, where $-\infty < \lambda_i \le 0,\ i = 1, 2, ..., k$, $\lambda_1 \oplus \lambda_2 \oplus ... \oplus \lambda_k = 0$. Here, as usual, for $x \in X$ by $\delta_x$ is denoted a functional on $C(X)$, defined by the formula $\delta_x(\varphi) = \varphi(x)$, $\varphi \in C(X)$, and called the Dirac measure. It is concentrated at the point $x$.

A closed set $A$ of a space $X$ is called a $Z$-set in $X$, if the identity map $id_X$ can be arbitrarily closely approximated by $f : X \to X \setminus A$. A countable union of $Z$-sets in $X$ is called a $\sigma$-$Z$-set in $X$.

**Theorem 1.** *For an arbitrary compact $X$ and every natural $n \in \mathbb{N}$, $n < |X|$, the subspace $I_n(X)$ is $Z$-set in $I(X)$. Therefore, $I_\omega(X)$ is a $\sigma$-$Z$-set.*

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## TO NUMERICAL MODELING OF A NONLINEAR PROBLEM OF THERMAL CONDUCTIVITY UNDER NONLINEAR BOUNDARY CONDITIONS OF THE KIND


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The nonlinear heat conduction problem described by a system of nonlinear differential equations [1-2] is considered, in the field $\Omega = \{\, r_0 \leqslant r \ < R;\ t_0 \leqslant t \leqslant T\,\}$

$$\frac{\partial u_k}{\partial t} = \lambda_k a_{k1} \frac{\partial}{\partial r}\left(a_{k2}\frac{\partial u_k}{\partial r}\right) \ + q_k(r,t) f_k(r,t,u_1,u_2) \tag{1}$$

with initial

$$u_k(t_0, r) = \psi_{k1}(r), \tag{2}$$

and nonlinear boundary conditions of the first kind

$$u_k(r_0,t) = \varphi_{k1}(u_k),\ \ u_k(R,t) = \varphi_{k2}(u), \tag{3}$$

where $a_{k\,i} = a_{k\,i}(u_1, u_2, u_{1r}, u_{2r}, r, t) > 0$, $q_k$ , $f_k$ , $\varphi_j-$ in the general case, are nonlinear functions dependent on $u_k$; $\lambda_k$ - heat conductivity coefficient, where $k = 1,2;\ \ i = 1,2;\ \ j = \overline{1,4}$. Due to the generality of the form of these functions, it is possible to obtain various linear and nonlinear boundary value problems of the first kind, which arise, in particular, in thermal and electrical calculations of thermal emission converters [1].

In the present work, thermal and electric fields are modeled using splitting methods. The linearization of nonlinear terms in (1) - (3) is performed by the methods of Picard and Newton [2].

Computational experiments were conducted on a test example of one model problem (1) - (3). The obtained numerical solutions, using explicit and implicit difference schemes, using iteration and with the initial data, show the acceptability of these methods. Practically acceptable numerical results were obtained using the developed algorithm and program.

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## ON HUA LO-KEN AND LAPLACE OPERATORS


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1. Consider space $\mathbb{C}^{mk}$. Points $z \in \mathbb{C}^{mk}$ we write in the form

$$z = (z_{11}, z_{12}, ..., z_{1k}; ...; z_{m1}, z_{m2}, ..., z_{mk}),$$

where $z_{\mu\nu} = x_{\mu\nu} + iy_{\mu\nu}$, $\mu = 1, 2, ..., m$; $\nu = 1, 2, ..., k$.

In some questions, points of $\mathbb{C}^{mk}$ space are conveniently represented as elements of $[m \times k]$ matrices:

$$\begin{pmatrix} z_{11} & z_{12} \dots z_{1k} \\ . & . \; \dots \; . \\ z_{m1} & z_{m2} \dots z_{mk} \end{pmatrix} = Z.$$

Then an isomorphism holds: $\mathbb{C}^{mk} \simeq \mathbb{C}[m \times k]$, where $\mathbb{C}[m \times k]$ is the set of $[m \times k]$-matrices.

Let

$$\partial_Z = \begin{pmatrix} \frac{\partial}{\partial z_{11}} & \frac{\partial}{\partial z_{12}} \cdots \frac{\partial}{\partial z_{1k}} \\ . & . \; \dots \; . \\ \frac{\partial}{\partial z_{m1}} & \frac{\partial}{\partial z_{m2}} \cdots \frac{\partial}{\partial z_{mk}} \end{pmatrix}, \quad \bar{\partial}_Z = \begin{pmatrix} \frac{\partial}{\partial \bar{z}_{11}} & \frac{\partial}{\partial \bar{z}_{12}} \cdots \frac{\partial}{\partial \bar{z}_{1k}} \\ . & . \; \dots \; . \\ \frac{\partial}{\partial \bar{z}_{m1}} & \frac{\partial}{\partial \bar{z}_{m2}} \cdots \frac{\partial}{\partial \bar{z}_{mk}} \end{pmatrix}$$

be differential operators (see [1], p. 116).

Consider in neighborhood $U$ of point $z \in \mathbb{C}^{mk}$ the function

$$f = f(z_{11}, z_{12}, ..., z_{1k}; ...; z_{m1}, z_{m2}, ..., z_{mk})$$

from class $C^1(U)$.

**Definition.** We say that $f(Z)$ is a holomorphic function in $U$, if $\bar{\partial}_Z f = 0$ in $U \subset \mathbb{C}^{mk}$.

Notice, that

$$\bar{\partial}_Z f = 0 \;\; \Leftrightarrow \;\; \frac{\partial f}{\partial \bar{z}_{\mu\nu}} = 0, \;\; \mu = 1, 2, ..., m; \;\; \nu = 1, 2, ..., k.$$

Let $E$ an open set in $\mathbb{C}^{mk}$ and $f \in C^2(E)$. Classic Laplacian define as usual

$$\Delta f = \sum_{\mu=1}^{m} \sum_{\nu=1}^{k} \left( \frac{\partial^2 f}{\partial x_{\mu\nu}^2} + \frac{\partial^2 f}{\partial y_{\mu\nu}^2} \right) = 4 \sum_{\mu=1}^{m} \sum_{\nu=1}^{k} \frac{\partial^2 f}{\partial z_{\mu\nu} \partial \bar{z}_{\mu\nu}}, \;\; \bar{z}_{\mu\nu} = x_{\mu\nu} - i y_{\mu\nu}, \;\; \mu = \overline{1, m}; \;\; \nu = \overline{1, k}.$$

2. Consider the Hua Loo-ken operator in the classical Cartan domain of the first type

$$K_1 = \{ Z \in \mathbb{C}[m \times k] : \;\; I^{(m)} - Z Z^* > 0 \},$$

where $I^{(m)}$ as usual, unit matrix of an order $m$, $Z^* = \bar{Z}'$ is a complex conjugate matrix with the transposed matrix $Z'$ ($H > 0$ for the Hermitian matrix $H$ means that $H$ is positively definite, i.e. all eigenvalues are positive).

The Hua Lo-Ken operator $Sp\Delta_Z$ has the following form (see [1], p. 117):

$$Sp\Delta_Z = \sum_{\mu,\nu=1}^{m} \sum_{\alpha,\beta=1}^{k} \left( \delta_{\mu\nu} - \sum_{\gamma=1}^{k} z_{\mu\gamma} \bar{z}_{\nu\gamma} \right) \left( \delta_{\alpha\beta} - \sum_{\delta=1}^{k} \bar{z}_{\delta\alpha} z_{\delta\beta} \right) \frac{\partial^2}{\partial z_{\mu\beta} \partial \bar{z}_{\nu\alpha}}.$$

Note that if $m = 1$, the domain $K_1$ coincides with the unit ball $B$ of $\mathbb{C}^k$, then the Hua Lo-Ken operator coincides with the invariant Laplace operator for $B$ (see [2], p. 54).

Let $\varphi_P$ be automorphism of the domain $K_1$, which takes the point $P \in K_1$ to 0

$$W = Q \left( Z - P \right) \left( I^{(k)} - P^* Z \right)^{-1} R^{-1}, \tag{1}$$

where $Q$ is the matrix of an order $m$ , $R$ is the matrix of an order $k$.

Now we formulate the chain rule, and it can be useful for searching complex derived matrix functions.

Let $E$ be an open set from $K_1$.

**Proposition**. *If $f \in C^2(E)$ and map $\varphi_P = (\varphi_{11}, ..., \varphi_{\mu\eta})$, $\mu = \overline{1, m}$; $\eta = \overline{1, k}$, is an automorphism of $K_1$, which point 0 translates to $P \in E$, then*

$$\Delta f(P) = \sum_{i,j=1}^{m} \sum_{\beta,\gamma=1}^{k} (D_{i\gamma} D_{j\beta}^* f)(P) \sum_{\alpha=1}^{m} \sum_{\delta=1}^{k} [(D_{\alpha\delta} \phi_{ij})(0)(D_{\alpha\delta}^* \phi_{\beta\gamma})(0)]. \tag{2}$$

**Theorem 1.** *The Hua Lo-Ken operator $Sp\Delta_Z$ written as*

$$Sp\Delta f(P) = \Delta(f \circ \varphi_P)(0),$$

*where $\Delta$ is the classical Laplace operator.*

**Theorem 2.** *If $f \in C^2(E)$, $E$ is open set of $K_1$ and $W = \psi(Z)$ automorphism of $K_1$, then*

$$Sp\Delta(f \circ \psi)(Z) = Sp\Delta f \circ \psi(Z),$$

*where $W \in E$.*

For $m = 1$, Theorem 2 completely coincides with Theorem 4.1.2. from [2].


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## THE UNIFIED TRANSFORMATION METHOD FOR IBVP TELEGRAPH EQUATION ON GENERAL STAR GRAPH


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We consider metric graph which obtained by connecting $n$ finite $B_1, B_2, ..., B_n$ and $m$ semi infinite $B_{n+1}, B_{n+2}, ..., B_{n+m}$ bonds at one point, called to be vertex of the graph. We correspond the bonds $B_j$, $(j = \overline{1,n})$ to the intervals $(0, L_j)$ and the bonds $B_r$, $(r = \overline{n+1, n+m})$ to interval $(0, \infty)$ to define coordinates in each bond. Here vertex of the graph corresponds to 0 on each bond.

In each bond of the graph consider the initial boundary value problem (IBVP) for the telegraph equation

$$u_t^{(j)}(x,t) = \sigma^2 u_{xx}^{(j)}(x,t), \;\; x \in B_j, \, t > 0, \, (j = \overline{1, n+m}). \tag{1}$$

The initial conditions are given by:

$$u^{(j)}(x, 0) = u_0^{(j)}(x), \; x \in B_j, \, (j = \overline{1, n+m}), \tag{2}$$

boundary and the asymptotic conditions

$$u^{(j)}(L_j, t) = h_j(t), \; t \geq 0, \, (j = \overline{1,n}), \tag{3}$$

$$\lim_{x \to \infty} u^{(r)}(x,t) = 0, \, t \geq 0, \, (r = \overline{n+1, n+m}) \tag{4}$$

on finite and semi infinite bonds, respectively.

Moreover, we need to define the following vertex (gluing) conditions for connectivity of the graph

$$u^{(1)}(+0, \, t) = u^{(2)}(+0, \, t) = ... = u^{(n+m)}(+0, \, t), \tag{5}$$

$$\sum_{j=1}^{n+m} \delta_j^2 u_x^{(j)}(+0,\, t) = 0. \tag{6}$$

The last conditions usually called continuity and flux conservation (Kirchhoff) conditions on branching point of the graphs. We suppose, that initial data are smooth enough functions they satisfies the conditions (5)-(6) and compatibility condition $u_0^{(j)}(L_j) = h_j(0), (j = \overline{1,n})$.

We find the solution of the problem (1)–(6) using Fokas method. This method uses special generalization of the Fourier transformation, called the unified transformation method. To satisfy boundary and vertex conditions we obtained the system of linear algebraic equations, using so called global relations.

**Theorem.** *The solution of the IBVP problem has the form*

$$u^{(j)}(x,t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} e^{ikx-wt} u_0^{(j)}(k)dk-$$

$$-\frac{1}{2\pi}\int_{\partial D_R^-} e^{ikx-ikL_j-wt}\frac{\widehat{u}_0^{(j)}(k) - \widehat{u}_0^{(j)}(-k) - 2ik\sigma^2\widetilde{g}_0(w,t)}{A_j}dk-$$

$$-\frac{1}{2\pi}\int_{\partial D_R^+} e^{ikx-wt}\frac{e^{ikL_j}\widehat{u}_0^{(j)}(k) - e^{-ikL_j}\widehat{u}_0^{(j)}(-k) + 2ik\sigma^2 h_0^{(j)}(w,t)}{A_j}dk,\ (j = \overline{1,n}),$$

$$u^{(r)}(x,t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} e^{ikx-wt}\widehat{u}_0^{(r)}(k)dk-$$

$$-\frac{1}{2\pi}\int_{\partial D^+} e^{ikx-wt}\widehat{u}_0^{(r)}(-k)dk-$$

$$-\frac{1}{\pi}\int_{\partial D^+} e^{ikx-wt} ik\sigma^2\widetilde{g}_0(w,t)dk,\ (r = \overline{n+1, n+m}).$$

*Here*

$$ik\sigma^2\widetilde{g}_0(w,t) = \frac{1}{\sum_{j=1}^{n}\delta_j^2\frac{B_j}{A_j} + \sum_{r=n+1}^{n+m}\delta_r^2}\cdot$$

$$\cdot\left[\sum_{j=1}^{n}\frac{\delta_j^2}{A_j}\left[e^{ikL_j}\widehat{u}_0^{(j)}(k) - e^{-ikL_j}\widehat{u}_0^{(j)}(-k) + 2ik\sigma^2 h_0^{(j)}(w,t)\right] + \sum_{r=n+1}^{n+m}\delta_r^2\widehat{u}_0^{(r)}(k)\right],$$

$$A_j = e^{ikL_j} - e^{-ikL_j},\ B_j = e^{ikL_j} + e^{-ikL_j}, (j = \overline{1,n}).$$

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# LOCAL DERIVATIONS OF THREE-DIMENSIONAL SOLVABLE LEIBNIZ ALGEBRAS

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The notions of local derivations were first introduced in 1990 by R.V.Kadison [1] and D.R.Larson,A.R.Souruor [2]. Investigation of local derivations on algebras of measurable operators were initiated in several papers. The main problems concerning this notion are to find conditions under which local derivations become derivations and to present examples of algebras with local derivations that are not derivations. Later, similar notions and corresponding problems have been devoted for non-associative algebras, for example Lie and Leibniz algebras. In [3] Sh.A.Ayupov and K.K.Kudaybergenov have proved that every local derivation on semi-simple Lie algebras is a derivation and gave examples of nilpotent finite-dimensional Lie algebras with local derivations which are not derivations. In this work local derivations of some three-dimensional solvable Leibniz algebras are investigated.

**Definition 1.** [4]. A vector space $L$ over a field $F$ with a binary operation $[-,-]$ is a (right) Leibniz algebra, if for any $x, y, z \in L$ the so-called Leibniz identity

$$[x,[y,z]] = [[x,y],z] - [[x,z],y]$$

holds.

Every Lie algebra is a Leibniz algebra, but the bracket in a Leibniz algebra needs not to be skew-symmetric.

For a Leibniz algebra $L$ consider the following central lower and derived sequences:

$$L^1 = L, \quad L^{k+1} = [L^k, L^1], \quad k \geq 1,$$

$$L^{[1]} = L, \quad L^{[k+1]} = [L^{[k]}, L^{[k]}], \quad k \geq 1,$$

**Definition 2.** A Leibniz algebra $L$ is called nilpotent, if there exists $p \in N$ such that $L^p = 0$. A Leibniz algebra $L$ is called solvable, if there exists $q \in N$ such that $L^{[q]} = 0$.

Note that any nilpotent Leibniz algebra is solvable Leibniz algebra.

Now we define the notion of derivation and local-derivation for Leibniz algebras.

**Definition 3.** A derivation on a Leibniz algebra $L$ is a linear map $d : L \to L$ which satisfies the Leibniz rule:

$$d([x,y]) = [d(x),y] + [x,d(y)]$$

for any $x, y \in L$.

**Definition 3:** A linear operator $\Delta$ is called a local derivation if for any $x \in L$, there exists a derivation $d_x : L \to L$ (depending on $x$) such that $\Delta(x) = d_x(x)$.

The following algebras are 3-dimensional solvable Leibniz algebras which are not nilpotent [5]:

$L_1 : [e_1, e_2] = e_1, \quad [e_2, e_1] = -e_1;$

$L_2 : [e_1, e_3] = e_1, \quad [e_2, e_3] = \alpha e_2, \quad [e_3, e_1] = -e_1, \quad [e_3, e_2] = -\alpha e_2, \quad \alpha \in C - 0;$

$L_3 : [e_1, e_3] = e_1 + e_2, \quad [e_2, e_3] = e_2, \quad [e_3, e_1] = -e_1 - e_2, \quad [e_3, e_2] = -e_2;$

$L_4 : [e_1, e_3] = e_1;$

$L_5 : [e_1, e_3] = \alpha e_1, \quad [e_2, e_3] = e_2, \quad [e_3, e_2] = -e_2, \quad \alpha \in C - 0;$

$L_6 : [e_2, e_3] = e_2, \quad [e_3, e_2] = -e_2, \quad [e_3, e_3] = e_1;$

$L_7 : [e_1, e_3] = 2e_1, \quad [e_2, e_2] = e_1, \quad [e_2, e_3] = e_2, \quad [e_3, e_2] = -e_2, \quad [e_3, e_3] = e_1;$

$L_8 : [e_1, e_2] = \beta e_1, \quad [e_2, e_3] = e_2, \quad \beta \in C - 0;$

$L_9 : [e_1, e_3] = e_1 + e_2, \quad [e_2, e_3] = e_2.$

In the following propositions we formulate main results of this work.

**Proposition 1.** *Any local derivation $\Delta$ on algebras $L_1, L_2, L_4, L_5, L_6, L_8$ is a derivation.*

**Proposition 2.** *There exist a local derivation on algebras $L_3, L_7, L_9$ which is not a derivation.*

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# CLASSIFICATION OF SOLVABLE LEIBNIZ ALGEBRAS WITH FILIFORM NILRADICAL LENGTH $n-1$ WITH NILRADICAL $M_2$


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Leibniz algebras were introduced by J.H.Lodas as a non- antisymmetric generalization of Lie algebras. Active investigations on Leibniz algebras theory show that many results of the theory of Lie algebras can be extended to Leibniz algebras. The analogue of Levi‘s theorem for Leibniz algebras is proven by D.Barnes and it is shown than any finite-dimensional complex Leibniz algebra is decomposed into semi-direct sum of semi-simple Lie algebra and solvable radical. Thus, the class of solvable algebras are very important in the varieaty of finite-dimensional Leibniz algebras. Note that solvable Leibniz algebras can be described by means of nilradical and its derivation. In this work the classification of solvable Leibniz algebras with filiform nilradical of length $n-1$ is obtained.

**Definition 1.** An algebra $(L, [-,-])$ over a field $F$ is called Leibniz algebra if for any $x, y, z \in L$ the so-called Leibniz identity:

$$[x,[y,z]] = [[x,y],z] - [[x,z,],y]$$

holds.

**Definition 2.** A linear map $d : L \to L$ of a Liebniz algebra $(L, [-,-])$ is said to be a *derivation* if for all $x, y \in L$ the following condition holds:

$$d([x,y]) = [d(x),y] + [x,d(y)]$$

For any Leibniz algebra $L$, we define following series:

$$L^1 = L, \quad L^{k+1} = [L^k, L], \quad k \geq 1$$

$$L^{[1]} = L, \quad L^{[s+1]} = [L^{[s]}, L^{[s]}], \quad s \geq 1$$

are said to be the *lower central* and *derived series* of $L$, respectively.

**Definition 3.** A Liebniz algebra $(L, [-,-])$ is said to be *nilpotent* (respectively, *solvable*), if there exists $n \in N$ $(m \in N)$ such that $L^n = 0$ (respectively, $L^{[m]} = 0$). The minimal number $n$ (respectively, $m$) with such property is said to be the *index of nilpotency*(respectively, *index of solvability*) of the algebra $L$.

**Definition 4.** An $n$-dimensional Leibniz algebra $L$ is said to be *filiform* if $\dim L^i = n - i, \ 2 \leq i \leq n$.

Let $L$ be a $Z$-graded Leibniz algebra with a finite number of non zero subspaces, i.e. $L = \bigoplus\limits_{i \in Z} V_i$, where $[V_i, V_j] \subseteq V_{i+j}$ for any $i, j \in Z$. We say that a Leibniz algebra $L$ admits a *connected gradation* if $L = V_{k_1} \oplus V_{k_2} \oplus \cdots \oplus V_{k_t}$, where each $V_i$ is non-zero for $k_1 \leq i \leq k_t$.

The number of subspaces $l(\oplus L) = k_t - k_1 + 1$ is called the length of the gradation. The length $l(L)$ of a Leibniz algebra $L$ is defined as

$$l(L) = \max\{l(\oplus L) = k_t - k_1 + 1 \mid L = V_{k_1} \oplus V_{k_2} \oplus \cdots \oplus V_{k_t} \quad \text{is a connected gradation.}\}$$

In the following theorem all filiform Leibniz algebras of length $n-1$ are described.

**Theorem 1.** [2] *Any $n$-dimensional filiform Leibniz algebra of (n − 1) is isomorphic to one of the following pairwise non-isomorphic algebras:*

$$NGF_2 = \begin{cases} [y_1, y_1] = y_3, \\ [y_i, y_1] = y_{i+1}, \quad 3 \leq i \leq n-1. \end{cases} \qquad M_1(k) = \begin{cases} [y_i, y_1] = y_{i+1} & 1 \leq i \leq n-2, \\ [y_i, y_n] = y_{k+i-j} & 1 \leq i \leq n-k \\ & 3 \leq k \leq n-1. \end{cases}$$

$$M_2(\alpha) = \begin{cases} [y_i, y_1] = y_{i+1}, & 1 \leq i \leq n-2 \\ [y_i, y_n] = y_{\frac{i+1}{2}+i-1}, & 1 \leq i \leq \frac{n-1}{2}, \\ [y_n, y_n] = \alpha y_{n-1}, & \alpha \neq 0 \end{cases} \qquad M_3 = \begin{cases} [y_i, y_1] = y_{i+1}, & 1 \leq i \leq n-2, \\ [y_n, y_n] = y_{n-1} \end{cases}$$

*where all omitted products are equal to zero and $\{y_1, y_2, \ldots, y_n\}$ is a basis of the corresponding algebra and $n$ is odd in the algebra $M_2(\alpha)$.*

In this work we describe solvable Leibniz algebras which nilradical is isomorphic to filiform Leibniz algebra $M_2(\alpha)$. For this purpose, fist we give the description of derivation of this algebra.

**Proposition.** *Any derivation of the algebra $M_2(\alpha)$ has the following matrix form:*

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} & \ldots & a_{1,n-2} & a_{1,n-1} & 0 \\ 0 & 2a_{11} & a_{12} & a_{13} & a_{14} & & a_{1,n-3} & a_{1,n-2} & 0 \\ 0 & 0 & 3a_{11} & a_{12} & a_{13} & \ldots & a_{1,n-4} & a_{1,n-3} & 0 \\ 0 & 0 & 0 & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & 0 & \ldots & \ldots & \ldots & (n-2)a_{11} & a_{12} & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & (n-1)a_{11} & 0 \\ 0 & 0 & 0 & 0 & 0 & \ldots & 0 & a_{n,n-1} & \frac{n-1}{2}a_{11} \end{pmatrix}$$

In the following theorem we prove that up to isomorphic there exist unique solvable Leibniz algebra with nilradical $M_2(\alpha)$.

**Theorem 2.** *Let $R$ be a solvable Leibniz algebra with nilradical $M_2(\alpha)$. Then $\alpha = 1$ and $R$ is isomorphic to the following algebra:*

$$\begin{cases} [y_i, y_1] = y_{i+1}, \quad 1 \le i \le n-2, & [y_i, y_n] = y_{\frac{i+1}{2}-i+1}, \quad 1 \le i \le \frac{n-1}{2} \\ [y_n, y_n] = y_{n-1}, & [y_i, x] = iy_i, \quad 1 \le i \le n-1, \\ [y_{n-1}, x] = y_{n-1}, & [y_n, x] = \frac{n-1}{2}y_n, \\ [x, y_1] = -y_1, & [x, y_n] = -y_{\frac{n-1}{2}} - \frac{n-1}{2}y_n \end{cases}$$

*where $\{x, y_1, y_2, \ldots, y_n\}$ is a basis of $R$*

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# CLASSIFICATION OF 11-DIMENSIONAL COMPLEX FILIFORM ALGEBRAS LEIBNIZ


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Leibniz algebras present a non commutative analogue of Lie algebras and they were introduced by J.-L.Loday [1], as algebras which satisfy the following identity:

$$[x,[y,z]] = [[x,y],z] + [[x,z],y].$$

A derivation of Leibniz algebra $L$ is a linear transformation, such that

$$d([x,y]) = [d(x),y] + [x,d(y)],$$

for any $x, y \in L$.

Pre-derivations of Leibniz algebras are a generalization of derivations which defined as follows.

**Definition 1.** A linear transformation $P$ of the Leibniz algebra $L$ is called a pre-derivation if for any $x, y, z \in L$,

$$P([[x,y],z]) = [[P(x),y],z] + [[x,P(y)],z] + [[x,y],P(z)].$$

For the given Leibniz algebra $L$ we consider the following central lower series:

$$L^1 = L, \ L^{k+1} = [L^k, L^1] \ k \geq 1.$$

**Definition 2.** A Leibniz algebra $L$ is called nilpotent if there exists $s \in N$ such that $L^s = 0$.

A nilpotent Leibniz algebra is called characteristically nilpotent if all its derivations are nilpotent. We say that a Leibniz algebra is strongly nilpotent if any pre-derivation is nilpotent. Since any derivation of the Leibniz algebra is a pre-derivation, it implies that a strongly nilpotent Leibniz algebra is characteristically nilpotent. An example of characteristically nilpotent, but nonstrongly nilpotent Leibniz algebra could be find in [3].

**Definition 3.** A Leibniz algebra $L$ is said to be filiform if $dimL^i = n - i$, where $n = dimL$ and $2 \leq i \leq n$.

From [3] we have that any complex 11-dimensional filiform Leibniz algebra admits a basis $\{e_1, e_2, \ldots, e_{11}\}$ such that the table of multiplication of the algebra has one of the following forms:

$$F_1(\alpha_4, \ldots, \alpha_{11}, \theta) : \begin{cases} [e_1,e_1] = e_3, \ [e_i,e_1] = e_{i+1}, & 2 \leq i \leq 10, \\ [e_1,e_2] = \sum\limits_{t=4}^{10} \alpha_t e_t + \theta e_{11}, & \\ [e_j,e_2] = \sum\limits_{t=j+2}^{11} \alpha_{t-j+2} e_t, & 2 \leq j \leq 9. \end{cases}$$

$$F_2(\beta_4, \ldots, \beta_{11}, \gamma) : \begin{cases} [e_1,e_1] = e_3, \quad [e_2,e_2] = \gamma e_{11}, & \\ [e_i,e_1] = e_{i+1}, & 3 \leq i \leq 10, \\ [e_1,e_2] = \sum\limits_{i=4}^{11} \beta_i e_i, & \\ [e_j,e_2] = \sum\limits_{t=j+2}^{11} \beta_{t-j+2} e_t, & 3 \leq j \leq 9. \end{cases}$$

omitted products are zero.

**Theorem 2.** *Every 11-dimensional strongly nilpotent filiform Leibniz algebra is isomorphic to one of the following pairwise non-isomorphic algebras:*

$F_1(1,-2,5,-14,42,-132,0,\alpha_{11},\theta)$, $F_1(0,0,1,0,0,-4,0,0,1)$,
$F_1(1,-2,5,-14,42,-132,429,-1430,0)$, $F_1(0,0,0,0,1,0,0,0,1)$,
$F_1(0,0,1,0,0,-4,1,\alpha_{11},\theta)$, $F_1(0,0,1,0,0,-4,0,1,\theta)$,
$F_1(1,-2,5,-14,42,-132,429,0,\theta)$, $F_1(0,0,1,0,0,-4,0,0,0)$,
$F_1(0,0,0,0,1,0,1,\alpha_{11},\theta)$, $F_1(0,0,0,0,1,0,0,1,\theta)$,
$F_1(1,-2,5,-14,42,-132,429,-1430,-1430)$, $F_1(0,0,0,0,1,0,0,0,0)$,
$F_1(0,1,0,0,0,\alpha_9,0,\alpha_{11},\theta)$, $F_1(0,1,0,-3,0,0,0,\alpha_{11},\theta)$,
$F_1(0,1,0,-3,0,12,0,0,\theta)$, $F_1(0,1,0,-3,0,12,0,-55,0)$,
$F_1(0,1,0,-3,0,12,0,-55,-55)$, $F_1(0,0,0,1,0,1,0,\alpha_{11},\theta)$,
$F_1(0,0,0,1,0,0,0,0,\theta)$, $F_1(0,0,0,1,0,0,0,-5,0)$,
$F_1(0,0,0,1,0,0,0,-5,5)$, $F_1(0,0,0,0,0,1,0,1,\theta)$,
$F_1(0,0,0,0,0,1,0,0,1)$, $F_1(0,0,0,0,0,1,0,0,0)$,
$F_1(0,0,0,0,0,0,0,1,0)$, $F_1(0,0,0,0,0,0,0,1,1)$,
$F_1(0,0,0,0,0,0,0,0,1)$, $F_1(0,0,0,0,0,0,0,0,0)$,

$F_1(0,1,0,-3,0,12,1,\alpha_{11},\theta)$, $F_1(0,1,0,-3,0,12,0,0,\theta)$,
$F_1(0,1,0,-3,0,12,0,-55,0)$, $F_1(0,1,0,-3,0,12,0,-55,-55)$,
$F_1(0,1,0,-3,0,12,0,\alpha,-55)$, $F_1(0,1,0,-3,0,12,0,\alpha,\alpha)$,
$F_1(0,0,0,1,0,0,1,\alpha_{11},\theta)$, $F_1(0,0,0,1,0,0,0,0,\theta)$,
$F_1(0,0,0,1,0,0,0,-5,0)$, $F_1(0,0,0,1,0,0,0,-5,-5)$
$F_1(0,0,0,1,0,0,0,\alpha,0)$, $F_1(0,0,0,1,0,0,0,\alpha,\alpha)$,
$F_1(0,0,0,0,0,1,1,\alpha_{11},\theta)$, $F_1(0,0,0,0,0,1,0,1,\theta)$,
$F_1(0,0,0,0,1,0,0,0,1)$, $F_1(0,0,0,0,1,0,0,0,0)$,

Similarly, we obtain that there exist 38 pairwise non-isomorphic strongly nilpotent filiform Leibniz algebras in the class $F_2(\beta_4,\ldots,\beta_{11},\gamma)$.

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# ON ASYMPTOTIC NORMALITY OF BRANCHING PROCESSES WITH INCREASING AND DEPENDENT IMMIGRATION

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We consider a sequence of branching processes with immigration (SBPI) $W(k), k \geqslant 0, W(0) = 0$. It can be defined by two families of independent, nonnegative, integer-valued random variables $\{Y_{ki}, k, i \geqslant 1\}$ and $\{\varepsilon_k, k \geqslant 1\}$ recursively as

$$W(k) = \sum_{i=1}^{W(k-1)} Y_{ki} + \varepsilon_k, k \geqslant 1.$$

Suppose that $\{Y_{ki}, k, i \geqslant 1\}$is the sequence of identically distributed and the families $\{Y_{ki}, k, i \geqslant 1\}$ and $\{\varepsilon_k, k \geqslant 1\}$ are independent. The variable $Y_{ki}$ can be interpreted as the number of offspring of the $i-$th individual in the $(k-1)-$th generation and $\varepsilon_k$ as the number of immigrating individuals to the $k-$th generation. In this interpretation $W(k)$ can be considered as the size of $k-$th generation of the population.
Set $a = \mathrm{E}Y_{11}$. Process $W(k)$ is called subcritical, critical or supercritical depending on $a < 1$, $a = 1$ or $a > 1$ respectively. There have been many research works on the limit theorems of SBPI. For instance, Rahimov [1,2] considered subcritical, critical and supercritical SBPI and proved central limit theorems for such processes. Our aim is to prove central limit theorem for subcritical branching processes with dependent immigration. We assume that $a < 1$ and $b = VarY_{11} < \infty$. We also assume that $\alpha(k) = \mathrm{E}\varepsilon_k < \infty$ and $\beta(k) = Var\varepsilon_k < \infty$ for each $k \geqslant 1$, where $\{\alpha(k), k \geqslant 1\}$, $\{\beta(k), k \geqslant 1\}$ are regularly varying functions with exponents $\alpha \geqslant 0$ and $\beta \geqslant 0$ respectively. Then $A(a,n) = \mathrm{E}W(n)$, $B^2(a,n) = VarW(n)$ are finite for each $n \geqslant 1$. Denote

$$Z(n) = \frac{W(n) - A(a,n)}{B(a,n)}, n \geqslant 1, \Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{u^2}{2}} du.$$

Our main result is the following theorem.

**Theorem.** *Assume that $\{\varepsilon_k, k \geqslant 1\}$ is a sequence of $m-$dependent random variables with $\alpha(n) \to \infty$ and $\max\limits_{1 \leqslant k \leqslant n} \beta(k) = o\left(\sum\limits_{k=1}^{n} a^{n-k} \alpha(k)\right)$. Then $\mathrm{P}(Z(n) < x) \Rightarrow \Phi(x)$ as $n \to \infty$, where $\Rightarrow$denotes convergence of distributions.*

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# DOUBLE-RELAXATION SOLUTE TRANSPORT IN POROUS MEDIA

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Solute transport in one-dimensional media can be described by the equation

$$m\frac{\partial c}{\partial t} + \frac{\partial F}{\partial x} = 0, \tag{1}$$

where $m$ – the porosity of the medium, $c$ – the concentration of solute dissolved in the filtrating liquid, $F$ – the total solute mass flow, consisting of the convection ($F_c$) and diffusion ($F_d$) parts, $F = F_c + F_d$, $t$ – time, $x$ – space coordinate.

Convective flow has the following form [1]

$$F_c = v_f c = vmc, \tag{2}$$

where $v_f$ – filtration velocity, $v$ – the physical fluid velocity.

Diffusion law with double-relaxation we write as

$$F_d + \lambda_1 \frac{\partial F_d}{\partial t} = -Dm\left(1 + \lambda_2 \frac{\partial}{\partial t}\right)\frac{\partial c}{\partial x}, \tag{3}$$

where $\lambda_1, \lambda_2$ – the relaxation times, $D$ – diffusion coefficient, which we take as a constant. In more general form instead of diffusion coefficient one can use dispersion coefficient that depends on filtration velocity distribution [1].

From (1)-(3) we obtain

$$\left(1 + \lambda_1 \frac{\partial}{\partial t}\right)\left(\frac{\partial c}{\partial t} + v\frac{\partial c}{\partial x}\right) = D\left(1 + \lambda_2 \frac{\partial}{\partial t}\right)\frac{\partial^2 c}{\partial x^2}$$

or

$$\frac{\partial c}{\partial t} + \lambda_1 \frac{\partial^2 c}{\partial t^2} + v\frac{\partial c}{\partial x} + v\lambda_1 \frac{\partial^2 c}{\partial t \partial x} = D\frac{\partial^2 c}{\partial x^2} + \lambda_2 D\frac{\partial^3 c}{\partial t \partial x^2}. \tag{4}$$

In order to assess the relaxation effects on solute transport characteristics we pose the following problem. Let in the semi-infinite porous medium initially filled with pure (without solute) fluid since $t > 0$ inflows liquid with constant solute concentration $c_0$.

Then the initial and boundary conditions take the form

$$c(0,x) = 0, \ \ \frac{\partial c(0,x)}{\partial t} = 0\,, \ \ 0 \le x < \infty, \tag{5}$$

$$c(t,0) = c_0, \ \ c(t,\infty) = 0. \tag{6}$$

Equation (4) with (5), (6) is solved and influence of $\lambda_1$ and $\lambda_2$ on solute transport characteristics is shown.

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# SOLUTE TRANSPORT IN CYLINDRICAL POROUS MEDIA WITH FRACTAL STRUCTURE

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Studies of anomalous solute transport in recent years has greatly increased. This is primarily dictated by the relevance of the problem with applications in various industries and technology. On the other hand, there are many unsolved theoretical problems, in particular, the question of the influence of the anomalous nature of the solute transport on the hydrodynamic parameters is not fully clarified.[1]

In this work, a problem of anomalous solute transport in a cylindrical two-zone media is solved. The medium consists of two semi-infinite coaxial cylinders, the inside one of which has greater flow and diffusion properties (zone $\Omega_1\,\{0 \le x < \infty,\ 0 \le r \le a\}$), and external one – with relatively low properties ($\Omega_2\,\{0 \le x < \infty,\ a \le r \le b\}$)

In the zone $\Omega_1$ solute transport is described by the equation

$$\theta_m \frac{\partial c_m}{\partial t} + \theta_{im} \frac{\partial c_{im}}{\partial t} = \theta_m D_m \frac{\partial^\beta c_m}{\partial x^\beta} - \theta_m v_m \frac{\partial c_m}{\partial x}, \qquad (1)$$

where $c_m$ - average concentration in $\Omega_1$, $c_{im}$ – average substance concentration in zone $\Omega_2$.

$$c_{im} = \frac{2}{b^2 - a^2} \int_a^b r c_a(t, x, r) dr, \qquad (2)$$

$c_a$ - local concentration in $\Omega_2$ zone, $\theta_m$, $\theta_{im}$ - relative porosity coefficient of $\Omega_1$ and $\Omega_2$ zone, $D_m$ - diffusion coefficient of $\Omega_1$ zone, $v_m$ - average propagation velocity in $\Omega_1$ zone, $t$ - time.

Solute transport in the zone $\Omega_2$ is described by the diffusion equation of fractional order [2]

$$\frac{\partial c_a}{\partial t} = D_a \frac{1}{r^{\beta_1}} \frac{\partial^{\beta_1}}{\partial r^{\beta_1}} \left( r^{\beta_1} \frac{\partial^{\beta_1} c_a}{\partial r^{\beta_1}} \right),\ a < r < b,\ 0 < \beta_1 \le 1 \qquad (3)$$

where $D_a$ - diffusion coefficient in $\Omega_2$.

On the basis of computer experiments it is shown that decreasing of $\beta_1$ , leads to “fast diffusion” in the zone of $\Omega_2$ and to slower growth of the profiles $c_m$.

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# ON NUMBER OF EIGENVALUES BELOW THE LOWER BOUNDARY OF THE ESSENTIAL SPECTRUM OF PIO

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Self-adjoint partial integral operators (PIO) arise in the theory of Schrodinger operators[1]. Spectral properties of a discrete Schrodinger operator $H$ are tightly connected (see [1,2]) with that of partial integral operators which participate in the presentation of operator $H$.

Let $A$ be a linear self-adjoint operator in the Hilbert space $\mathcal{H}$. Essential spectrum and discrete spectrum of the operator $A$ are denoted by $\sigma_{ess}$ and $\sigma_{disc}$, respectively. We call the number $\mathcal{E}_{\min}(A)$ ($\mathcal{E}_{\max}(A)$) the lower (the higher) boundary of the essential spectrum of $A$.

Let $\Omega_1 = [a,b]^{\nu_1}, \Omega_2 = [c,d]^{\nu_2}$ and $k_0, k_1, k_2$ are continuous functions on $\Omega_1 \times \Omega_2$, $\Omega_1^2 \times \Omega_2$, $\Omega_1 \times \Omega_2^2$ respectively, $k_0$ is real function, $k_1(x,s,y) = \overline{k_1(s,x,y)}, k_2(x,t,y) = \overline{k_2(x,y,t)}$. We define the linear self-adjoint bounded operator $H$ in the Hilbert space $L_2(\Omega_1 \times \Omega_2)$ by rule $H = T_0 - (T_1 + T_2)$, where operators $T_0$, $T_1$ and $T_2$ are defined by the following formulas:

$$T_0 f(x,y) = k_0(x,y) f(x,y),$$

$$T_1 f(x,y) = \int_{\Omega_1} k_1(x,s,y) f(s,y) ds, \;\; T_2 f(x,y) = \int_{\Omega_2} k_2(x,t,y) f(x,t) dt,$$

We study the existence Efimov's effect in the model (1) in the case $\mathcal{E}_{\min}(H) \neq 0$. Consider this problem for the function $k_0(x,y)$ of the form $k_0(x,y) = u(x) + u(y)$. Let $u(x)$ and $v(y)$ is continuous nonnegative function on $\Omega_1$ and $\Omega_2$, respectively and suppose $k_1(x,s,y) = k_1(x,s)$, $k_2(x,t,y) = k_2(y,t)$. We define self-adjoint compact integral operators $Q_1 : L_2(\Omega_1) \to L_2(\Omega_1)$ and $Q_2 : L_2(\Omega_2) \to L_2(\Omega_2)$ by the following equalities

$$Q_1 \varphi(x) = \int_{\Omega_1} k_1(x,s) \varphi(s) ds, \quad Q_2 \psi(y) = \int_{\Omega_2} k_2(y,t) \psi(t) dt$$

and suppose that $Q_1 \geq \theta, \;\; Q_2 \geq \theta$.

Let $u(x) \geq 0, \;\; x \in \Omega_1, v(y) \geq 0, \;\; y \in \Omega_2$ and $u^{-1}(\{0\}) \neq \emptyset, \;\; v^{-1}(\{0\}) \neq \emptyset$.

**Theorem 1.** *Let be* $k_0(x,y) = u(x) + v(y)$ *and* $k_1(x,s,y) = k_1(x,s)$, $k_2(x,t,y) = k_2(y,t)$.

*a) for existence Efimov's effect in the model* (1) *it is necessary, that* $\dim(Ran(Q_1)) = \infty$ *or* $\dim(Ran(Q_2)) = \infty$;

*b) if* $\dim(Ran(Q_1)) < \infty$ *and* $\dim(Ran(Q_2)) < \infty$, *then the Efimov's effect is absence in the model* (1).

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## ON INEQUALITIES FOR MOMENTS OF BRANCHING PROCESSES WITH IMMIGRATION

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One of the main problems of probability theory is to estimate the central moments of random variables. It was obtained several estimates for the sum of moments of independent random variables (see for instance [1]). In this paper we present an estimate for the second central moments of branching processes with immigration.

Let $\{\xi_{k,i}, k, i \geq 1\}$ and $\{\varepsilon_k, k \geq 1\}$ be the sequences of independent, nonnegative, integer valued and identically distributed random variables. Suppose that the sequences $\{\xi_{k,i}, k, i \geq 1\}$ and $\{\varepsilon_k, k \geq 1\}$ are also independent.

We consider a sequence of branching processes with immigration $\{\xi_{k,i}, k, i \geq 1\}$ given by recursion

$$X_0,\ X_k = \sum_{j=1}^{X_{k-1}} \xi_{k,j} + \varepsilon_k,\ k \geq 1$$

where $X_0$ is an nonnegative, integer valued random variables and independent of $\{\xi_{k,i}, \varepsilon_k,\ k, i \geq 1\}$.

For a fixed $n \geq 1$ we can interpret $X_k$ as the size of $k-$th generation of a population, where $\xi_{k,j}$ is the number of offsprings of the $j-$th individual in the $(k-1)-$st generation and $\varepsilon_k$ is the number of immigrants contributing to the $k-$th generation.

Denote

$$m = E\xi_{1,1},\ \sigma^2 = D\xi_{1,1},\ \lambda = E\varepsilon_1,\ \ b^2 = D\varepsilon_1,\ \nu = EX_0.$$

**Theorem 1.** *Suppose that $m \neq 1$ and $b^2 < \infty,\ \sigma^2 < \infty$. Then*

$$C_1(p)[\sigma^2 \frac{m^{n-1}(m^n-1)}{m-1}(\nu + \frac{\lambda}{m-1}) + \frac{m^{2n}-1}{m^2-1}(b^2 - \frac{\sigma^2\lambda}{m-1})] \leq E|X_n - EX_n|^2 \leq$$

$$\leq C_2(p)[\sigma^2 \frac{m^{n-1}(m^n-1)}{m-1}(\nu + \frac{\lambda}{m-1}) + \frac{m^{2n}-1}{m^2-1}(b^2 - \frac{\sigma^2\lambda}{m-1})]$$

*where $C_1(p)$ and $C_2(p)$ are constants independent on $p$.*

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## SOME PROPERTIES OF $k$-CONVEX FUNCTIONS

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Class of $k-convex$ functions was studied by many mathematicians, namely Z. Błocki, N.S. Trudinger, X.-J. Wang. In particular, this result is the starting point of study of $k-convex$ functions ([3]). Furthermore, a non linear potential theory has been developed ([1], [3]). In this work we give a theorem about $(k-1)-convexity$ of $k-convex$ functions on hyperplanes.

Let $\Omega$ be a domain in $\mathbb{R}^n$. For $k = 1, ..., n$ and $u \in C^2(\Omega)$ the $k$-Hessian operator $F_k$ is defined by

$$F_k[u] = \sum_{i_1<...<i_k} \lambda_{i_1} \cdot ... \cdot \lambda_{i_k}$$

where $\lambda = (\lambda_1, ..., \lambda_n)$ denotes the vector of the eigenvalues of the Hessian matrix of second derivatives $D^2u$. Alternatively we may write

$$F_k[u] = [D^2u]_k$$

where $[A]_k$ denotes the sum of the $k \times k$ principal minors of an $n \times n$ matrix $A$.

Definition [1]. A function $u \in C^2(\Omega)$ is called $k - convex$ in $\Omega$ if $F_j[u] \geq 0$ for $j = 1, ..., k$.

When $k = 1$, we have $F_1[u] = \Delta u$ and $1 - convex$ functions are subharmonic. When $k = n$, $F_k[u] = \det D^2u$, the Monge-Ampère operator, and $n-convex$ functions are convex[1].

The main results of the work are the next

**Theorem.** *Let $u$ be a smooth function and $k$-convex on $\Omega$, then for any hyperplane $P \subset R^n$, the restriction $u|_P$ belongs to $(k-1)$-convex.*

**Corollary.** *Let $u$ be a smooth function and $k$-convex on $\Omega$, then for any plane $P \subset \mathbb{R}^n, \dim P = n - k + 1$, the restriction $u|_P$ is a subharmonic function.*

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## SOLUTE TRANSPORT WITH LONGITUDINAL AND TRANSVERSAL DIFFUSION IN A FRACTAL POROUS MEDIUM

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Fractional calculus is being applied to describe complex processes in different fields, such as biology, mechanics, etc. Many problems have been expressed and solved by fractional calculus, but there are still many unsolved problems.

Molecular transverse diffusion through heterogeneous medium is accounted for in solute mass transport originating from a uniform pulse-type stationary point source [1]. In this work, the corresponding two-dimensional fractional advection–dispersion equation with variable coefficients is solved by the explicit finite difference method. The heterogeneity of the medium is described by a position dependent linear non-homogeneous expression for velocity with unsteady exponential variation with time. Variation of the dispersion parameter due to heterogeneity is considered proportional to square of the velocity.

The solute transport with longitudinal and transversal diffusion is described by fractional advection-dispersion equation.

$$\frac{\partial c(x,y,t)}{\partial t} = \frac{\partial^{\beta_1}}{\partial x^{\beta_1}}\left(D_x(x,t)\cdot\frac{\partial c(x,y,t)}{\partial x} - u\left(x,t\right)\cdot c\left(x,y,t\right)\right) +$$

$$+\frac{\partial^{\beta_2}}{\partial y^{\beta_2}}\left(D_y(y,t)\cdot\frac{\partial c(x,y,t)}{\partial y} - v\left(y,t\right)\cdot c\left(x,y,t\right)\right) \qquad 0<\beta_1\le 1, \quad 0<\beta_2\le 1, \qquad (1)$$

where, $c(x,y,t)$ – is the solute concentration of the pollutant being transported along the flow field through the medium at a position $(x,y)$ at time $t$, $u(x,t)$ and $v(y,t)$ – are the flow velocities through longitudinal and transversal ($x$ and $y$) directions, respectively, $D_x(x,t)$ and $D_y(y,t)$ – are the diffusion coefficients, $\beta_1$ and $\beta_2$ – are the orders of fractional derivatives.

Equations (1) is solved for given initial and boundary conditions and it is shown that, decreasing the orders of derivatives ($\beta_1$ and $\beta_2$) leads to wider spreading of concentration profiles.

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## DESCRIPTION OF THE SET OF REGULAR DOUBLY STOCHASTIC OPERATORS

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Let $S^{m-1}$ and $V : S^{m-1} \to S^{m-1}$ be the $(m-1)$-dimensional simplex and a doubly stochastic operator (DSO), respectively. Namely, $V$ is continuous and $V(x) \prec x$ for any $x \in S^{m-1}$, where $\prec$ is classical majorization. By the properties of classical majorization and the definition of doubly stochastic operators it is immediately implied that the set of doubly stochastic operators is convex. Furthermore, this set is subset of finite dimensional vector space. Hence, extreme points of the set of doubly stochastic operators is finite.

A stochastic operator is called regular operator if it has unique fixed point and forward orbit of any initial point converges to this fixed point. If a stochastic operator is simultaneously doubly stochastic and regular , then this operator is called regular doubly stochastic operator (RDSO). It can be easily proved that the unique fixed point of any $(m-1)-$ dimensional RDSO is $(1/m, 1/m, ..., 1/m)$. The main characteristic feature of orbits of DSO is its convergence to some periodic orbit. By this statement and the definition of regular stochastic operators it is directly followed that a DSO is RDSO if and only if it has not periodic orbits. This is typical property of RDSOs. The following theorem describes more accurately the fixed and periodic points of the convex combination of DSOs.

**Theorem.** *Let $V_1, V_2, ..., V_t$ be the same dimensional doubly stochastic operators and $(\lambda_1, \lambda_2, ..., \lambda_t) \in intS^{t-1}$ . Then the following holds:*

*(i) $Fix(\sum_{i=1}^{t} \lambda_i V_i) = \bigcap_{i=1}^{t} Fix V_i$*

*(ii) for $\forall k \in N$, $Per_k(\sum_{i=1}^{t} \lambda_i V_i) \subset \bigcap_{i=1}^{t} Per_k V_i$.*

The proof of this theorem based on the geometric definition of majorization. The theorem implies the following corollary:

**Corollary.** *Let $V_1, V_2, ..., V_t$ be the same dimensional doubly stochastic operators*

*(i) If for any $k \geqslant 2$ $\bigcap_{i=1}^{t} Per_k V_i = \emptyset$, then for $\forall(\lambda_1, \lambda_2, ..., \lambda_t) \in intS^{t-1}$ every forward orbit of $\sum_{i=1}^{t} \lambda_i V_i$ converges.*

*(ii) If at least one of $V_1, V_2, ..., V_t$ is regular then for $\forall(\lambda_1, \lambda_2, ..., \lambda_t) \in intS^{t-1}$ the convex combination $\sum_{i=1}^{t} \lambda_i V_i$ is also regular. Hence the set of RDSOs is convex.*

*(iii) Any relative interior point (an operator) of the convex set of doubly stochastic operators is RDSO.*

Existence of a regular doubly stochastic operator is asked as an open problem in [1]. By the third part of the above corollary it is implied that the set of RDSOs is dense in the set of DSOs . Hence the third part of the above corollary is more general and full answer to the question.

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## STRUCTURAL-PARAMETRIC IDENTIFICATION OF FUZZY NEURAL NETWORKS BY RECURSIVE METHOD

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This paper proposes a structural-parametric identification method to models in the form of fuzzy rule based neural networks. The proposed method is a system identification method which includes a new recursive clustering method for the structure and parameter initialization. The algorithm solves the problem of initial structure and parameter identification and adjustments of membership functions. Numerical experiments based on data sets are presented to illustrate the advantages of the proposed method and shows effectiveness and accuracy of the proposed approach.

## ON THE DYNAMICS OF RATIONAL POINTS OF MAPS $f(x) = |2x-1| : [0,1] \to [0,1]$

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It is known that the dynamic properties of the mapping $f(x) = |2x-1|$ are well studied. If the starting point $x_0 \in [0,1] \setminus Q$, then the set of limit points is everywhere dense in [0,1]. In the case of $x_0 \in [0,1] \cap Q$ is a trajectory, either after finite number of iteration gets into the set of fixed points, or forms a periodic orbit. In this paper, we study the case $x_0 = \frac{n}{m}$ is depending on m. For several m, we obtain several independent orbits.

**Theorem 1.** *If $x_0 = \frac{k}{2^m}$ and $x_0 = \frac{k}{2\cdot 3^m}$ , where k,$m \in N$, then the orbit of a point in finite number of steps will become equal to a fixed point. In this case, the orbit is not periodic.*

**Corollary.** *If $x_0 \in [0,1] \setminus Q$ , then $\{x_n\}$ is dense in [0,1].*

**Corollary.** *If $x_0 \in [0,1] \setminus Q$ , then $\{x_n\}$ is uniformly distributed on [0,1].*

**Corollary.** *If $x_0 \in [0,1] \cap Q$ , then $\{x_n\}$ is always periodic.*

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**ESTIMATES OF THE BLOW-UP SOLUTION OF A CROSS-DIFFUSION PARABOLIC SYSTEM NOT IN DIVERGENCE FORM**

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Consider a nonlinear cross-diffusion parabolic system equations non-divergent form

$$\frac{\partial u_i}{\partial t} = u_{3-i}^{\alpha_i}\nabla\left(u_i^{m_i-1}\nabla u_i\right) + u_i^{p_i},\ i=1,2 \quad \text{в} \quad D\times(0,T) \tag{1}$$

$$\frac{\partial u_i}{\partial n} + \sigma_i(x,t)u_i = 0,\ i=1,2 \quad \text{на} \quad D\times(0,T) \tag{2}$$

$$u_i(x,0) = u_{0i}(x) > 0,\ i=1,2 \quad \overline{D} \tag{3}$$

where $D$ is smooth bounded domain in $R^N$, $\overline{D}$ is its closure, $N \geq 2$, $\nabla$ is gradient symbol, $\frac{\partial}{\partial n}$ is directional derivative in the external normal direction, $\sigma_i$ $(i=1,2)$ - non-negative function in $C^1(\bar{Q_T})$ $(Q_T = D\times(0,T),\ R^+ = (0,+\infty))$.

In this paper, we prove a theorem of upper bounds for unbounded (blow-up) solutions in a finite time.

**Theorem.** *Let $u$ be a solution $(C^3(D\times(0,T))\cap C^2(\overline{D}\times(0,T))$ to problem (1) - (3). Suppose that the following conditions are true:*
*1) At $s\in R^+$ $0\leq p_i - m_i \leq 1,\ \ 0\leq s\leq p_i-1$,*
*2) $\sigma_i(t)\geq 0$, $\sigma_{it}(t)\leq 0$ $\ t\in D\times(0,T)$,*
*3) $\beta_i = \min\limits_{\overline{D}}\left\{\frac{u_{i0}^{m_i-1}}{u_{i0}^{p_i}}\left[u_{3-i0}^{\alpha_i}\nabla\left(u_{i0}^{m_i-1}\nabla u_{i0}\right) + u_{i0}^{p_i}\right]\right\} > 0$,*
*4) $\int\limits_{M_{0i}}^{+\infty} s^{m_i-p_i-1}ds = \frac{M_{0i}^{m_i-p_i}}{p_i-m_i}, \qquad m_i < p_i$,*
*where $M_{0i} = \max\limits_{\overline{D}} u_{i0},\ i=1,2$.*
*Then $u_i(x,t), (i=1,2)$ blows up in finite time $T$, and the following estimate is valid $u_i(x,t)\leq (p_i-m_i)\,\beta_i^{\frac{1}{m_i-p_i}}(T-t)^{\frac{1}{m_i-p_i}}$, $T\leq \frac{1}{\beta_i}\frac{M_{oi}^{m_i-p_i}}{p_i-m_i}$, $i=1,2$.*

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## ONE APPROACH TO IMPLEMENTING A FUZZY GENETIC ALGORITHM

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We will consider the optimization problem, should minimize the function $f(X)$, $X \in S$, where $S \subseteq R^D$ and $D$ вЂ" dimension of decision variables. Each solution is designated as $X_{i,G} = [x_{1,i,G}, x_{2,i,G}, ..., x_{D,i,G}]$, where $i = 1, 2, ..., NP$, $NP$ is the size of the population and $G$ the number of the current iteration. Here is a description of a fuzzy genetic algorithm:

1. Initialization of the chromosome. The initial population generated randomly in a limited search space for a given minimum and maximum limits. $j$- th parameter $X_{i,G}$ is initialized as follows:

$$x_{i,j,G} = L_j + rndreal(0,1) \cdot (U_j + L_j)$$

where is the random number in the range $[0, 1]$ and $L_j$ and $U_j$ represent the lower and upper bounds of the corresponding variable.

2. Crossing over operator is applied with a certain probability so that recombination of the parent chromosomes and the creation of descendants did not result in incorrect solutions. Crossover operator is applied to each pair of the target vector. $X_{i,G}$ and the corresponding mutant vector $V_{i,G}$ to generate a trial vector

$$U_{i,G} \cdot u_{i,G} = \begin{cases} v_{i,j,G} \;\; if\, rndreal(0,1) \le C_r \;\; j = j_{rand} \\ x_{j,i,G} \text{ otherwise} \end{cases}$$

where $j_{rand}$ is an integer randomly selected from the range $[1, D]$.

3. The mutation is applied independently to both subchromosomes, with a random selection of a pair of genes in the chromosome, and they are exchanged between the subchromosomes;

4. As the selection method is used roulette wheel, chromosomes survive with a probability proportional to the value of their fitness function.

The selection operator is described as follows:

$$X_{i,G+1} = \begin{cases} U_{i,G} \;\; if\, f(U_{i,G}) \le f(X_{i,G}) \\ X_{i,G} \text{ otherwise} \end{cases}$$

The fuzzy logic controller used is a two-dimensional system in which there are two parameters $e_1$,$e_2$. Parameter values are determined by the formulas:

$$e_1(t) = \frac{f_{\max}(t) - f_{ave}(t)}{f_{\max}(t)},\; e_2(t) = \frac{f_{ave}(t) - f_{ave}(t-1)}{f_{\max}(t)}$$

# CRITERION OF $SO(n,p,C)$-EQUIVALENCE OF ELEMENTARY SURFACES

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Let $C^n$ be an $n$-dimensional linear space over the field of complex numbers $C$, and let $GL(n,C)$ be the group of all invertible linear transformations of the space $C^n$. Elements from $C^n$ are represented as $n$-dimensional vector-columns $\overrightarrow{x} = \{\overrightarrow{x}_j\}_{j=1}^n$, and the transformations $g \in GL(n,C)$ as $n \times n$-matrices $(g_{ij})_{i,j=1}^n$, where $x_i,\ g_{ij} \in C,\ i,j = 1,...,n$. The action $g \in GL(n,C)$ on the vector $\overrightarrow{x} = \{\overrightarrow{x}_j\}_{j=1}^n \in C^n$ is the multiplication of the matrix $g$ by the column-vector $\overrightarrow{x}$ (writing: $g\overrightarrow{x}$).

A $C^\infty$-differentiable mapping $x : (0,1) \times (0,1) \to C^n$ is called an elementary surface. If $G$ is a subgroup of $GL(n,C)$, then the two elementary surfaces $\overrightarrow{y}(s,t)$ and $\overrightarrow{x}(s,t)$ are called equivalent, if $\overrightarrow{y}(s,t) = g\overrightarrow{x}(s,t)$ for some $g \in G$ and any $s,t \in (0,1)$.

Let $O(n,p,C)$ be a pseudo-orthogonal subgroup of $GL(n,C)$, that is, $O(n,p,C) = \{g \in GL(n,C) : g^T e_p g = e_p\}$, where $g^T$-transposed matrix of $g$, and $e_p = (e_{ij}^p)_{i,j=1}^n$ is the matrix from $GL(n,C)$, for which $e_{ii}^p = 1$ if $i = 1,...,p$, $e_{ii}^p = -1$ if $i = p+1,...,n$, $e_{ij}^p = 0$ if $i \neq j$, $p \in 1,...,n-1$.

By $SO(n,p,C) = \{g \in O(n,p,C) : detg = 1\}$ we denote a special pseudo-orthogonal subgroup of $GL(n,C)$.

For each surface $\overrightarrow{x}(s,t) = (x_j(s,t))_{j=1}^n$ consider the $n \times n$-matrix $M_s(\overrightarrow{x}) = \left(\frac{\partial^{i-1} x_j(s,t)}{\partial s^{i-1}}\right)_{i,j=1}^n$, where $\frac{\partial^0 x_j(s,t)}{\partial s^0} = x_j(s,t)$ for all $j = 1,...,n$, and put $M_{ss}^{'}(\overrightarrow{x}) = \left(\frac{\partial^i x_j(s,t)}{\partial s^i}\right)_{i,j=1}^n$, $M_{st}^{'}(\overrightarrow{x}) = \left(\frac{\partial^i x_j(s,t)}{\partial s^{i-1}\partial t}\right)_{i,j=1}^n$.

A surface $\overrightarrow{x}(s,t)$ is called regular if the determinant $detM_s(\overrightarrow{x})(s,t) \neq 0$ for all $s,t \in (0,1)$.

The following theorem gives necessary and sufficient conditions for $SO(n,p,C)$-equivalence of regular surfaces $\overrightarrow{x}(s,t)$ and $\overrightarrow{y}(s,t)$.

**Theorem 1.** *Two regular surfaces $\overrightarrow{x}(s,t)$ and $\overrightarrow{y}(s,t)$ are $SO(n,p,C)$-equivalent if and only if for any $s,t \in (0,1)$ we have the following equalities:*

$$M_s^{-1}(\overrightarrow{x})(s,t) \cdot M_{ss}^{'}(\overrightarrow{x})(s,t) = M_s^{-1}(\overrightarrow{y})(s,t) \cdot M_{ss}^{'}(\overrightarrow{y})(s,t);$$

$$M_s^{-1}(\overrightarrow{x})(s,t) \cdot M_{st}^{'}(\overrightarrow{x})(s,t) = M_s^{-1}(\overrightarrow{y})(s,t) \cdot M_{st}^{'}(\overrightarrow{y})(s,t);$$

$$M_s^T(\overrightarrow{x})(s,t) \cdot e_p \cdot M_s(\overrightarrow{x})(s,t) = M_s^T(\overrightarrow{y})(s,t) \cdot e_p \cdot M_s(\overrightarrow{y})(s,t);$$

$$\det M_s(\overrightarrow{x})(s,t) = \det M_s(\overrightarrow{y})(s,t).$$

# HOLMGREN PROBLEM FOR GENERALIZED HELMHOLTZ EQUATION WITH THE THREE SINGULAR COEFFICIENTS

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Fundamental solutions for the equation

$$H_{\alpha,\beta,\gamma}(u) \equiv u_{xx} + u_{yy} + u_{zz} + \frac{2\alpha}{x}u_x + \frac{2\beta}{y}u_y + \frac{2\gamma}{z}u_z - \lambda^2 u = 0 \qquad (1)$$

are founded in the work [1] and one of them will be used at solving a boundary problem for equation (1). Here $0 < 2\alpha, 2\beta, 2\gamma < 1,\ \alpha, \beta, \gamma = const$.

Let $D \subset \mathrm{R}_3^+$ be a finite simple-connected domain bounded by planes

$$D_1 = \{(x, y, z) : x = 0,\ 0 < y < b,\ 0 < z < c\},$$

$$D_2 = \{(x, y, z) : y = 0,\ 0 < x < b,\ 0 < z < c\},$$

$$D_3 = \{(x, y, z) : z = 0,\ 0 < x < a,\ 0 < y < b\}$$

and by a surface $D_4$, which intersects with domains $D_i\ (i = 1, 2, 3)$. Here $a, b, c = const > 0$.

**Holmgren problem.** Find a function $u(x, y, z) \in C\left(\bar{\Omega}\right) \cap C^2(\Omega)$, satisfying equation (1) in $D$ and conditions

$$\left(x^{2\alpha}\frac{\partial u}{\partial x}\right)\bigg|_{x=0} = \nu_1(y, z),\ (y, z) \in D_1; \quad \left(y^{2\beta}\frac{\partial u}{\partial y}\right)\bigg|_{y=0} = \nu_2(x, z),\ (x, z) \in D_2;$$

$$\left(z^{2\gamma}\frac{\partial u}{\partial z}\right)\bigg|_{z=0} = \nu_3(x, y),\ (x, y) \in D_3; \quad u(x, y, z) = \varphi(x, y, z),\ (x, y, z) \in \bar{D}_4,$$

where $\nu_1$, $\nu_2$, $\nu_3$ and $\varphi$ are given functions, and, moreover, $\nu_1$, $\nu_2$ and $\nu_3$ can reduce to an infinity of the order less than $1 - 2\alpha_1 - 2\alpha_2 - 2\alpha_3$ on the boundaries of $D_1$, $D_2$ and $D_3$, respectively.

To prove the uniqueness of the solution, as usual, we suppose that the problem has two $u_1$ and $u_2$ solutions. Denoting $u = u_1 - u_2$ we get homogeneous Holmgren problem. Further we have to prove that the homogeneous problem has only trivial solution. In this case by applying famous formula of Gauss-Ostrogradsky one can easily get

$$\int\int\int_D x^{2\alpha} y^{2\beta} z^{2\gamma} \left[u_x^2 + u_y^2 + u_z^2 + \lambda^2 u^2\right] dxdydz = 0.$$

Hence, follows that $u_x = u_y = u_z = 0$, which implies $u(x, y, z) = const$. Considering homogeneous condition $u(x, y, z) = 0,\ (x, y, z) \in \bar{D}_4,$ conclude that $u(x, y, z) \equiv 0$ in $\bar{D}$.

The existence of the solution is proved by method of Green's functions. For this aim suppose that $a = b = c$ and $D_4$ is $\frac{1}{8}$ part of the sphere with origin on the point $O(0, 0, 0)$ and with radius $R = a$. In this case Green function for the Holmgren problem is represented by confluent hypergeometric function in four variables [1]. The solution of the Holmgren problem is found in explicit form.

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## USING COMPUTER TECHNOLOGIES IN THE HEALTH PROGRAM OF STUDENTS

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The individual aspect of health is closely related to the quantitative and qualitative approach to its assessment and the concept of the norm, which can be considered as the optimal range of fluctuations of indicators characterizing the structural and functional state of the organism, its organs and systems, within which this quality is maintained. The age aspect of health is determined by the fact that each stage of human development is characterized by its own specific features of relations with external (physical adaptation) and social (social adaptation) environments. This is due to the peculiarities of the deployment of the human genetic program itself in time and the nature of the requirements of society to a person in each age period of his development. That is, it is about the fact that for each age stage there must be their own criteria for health, determined by the morphofunctional organization peculiar to this age. The student period is characterized by the presence of a large number of physical and psycho-emotional stress. The degree to which the teacher can correctly determine the level of the psychophysical state of the student depends on his further health and the level of self-preparation. Before the start of regular classes, first-year students undergo a medical health screening to identify diseases, determine levels of health and physical fitness. In accordance with one of the levels of health, students are divided into groups. Formed groups of students are actively involved in the educational process, which helps first-year students to more quickly integrate into the student environment, adapt to new educational conditions, and sometimes solve psychological problems of adaptation at the university as a whole. The use of team sports in physical education classes allows the teacher to gradually and differentially give loads to students with low levels of health and below average. At the same time, special attention is paid to the game technique, to attracting students to the development of individual plans.

The practice of group games has shown that physical fitness raises the level of physical health, respectively, improves performance: heart rate (pulse beats / min); Rufie test (for assessing the health - heart rate); breath-holding time during inhalation and exhalation (Stange and Genchi tests, sec.); blood pressure (mm.); lung capacity (ml.); with the subsequent calculation of a vital indicator and a double product. So, if at the beginning of the school year at the first measurement of the Rufíe test, the average indicator for first-year girls is 11.8, by the end of the first course - 5-7. Samples of Stage and Genchi change from 35 to 58 (s), respectively.

# MATHEMATICAL MODEL OF OIL FILTERING TAKING INTO ACCOUNT THE PRECIPITATION OF GEL PARTICLES IN POROUS MEDIA

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Investigation of oil filtration processes in porous media indicates that, as a result of the regular operation of the mines, there is a tendency of small particles to be deposited on the pores. This effectively affects oil filtration processes in porous media. Therefore, the mathematical model of the calculation of the main indicators of oilfields is influenced by the change in the porosity and permeability coefficients by time, taking into account the movement of the particles. The mathematical model of non-homogeneous one-dimensional non-phosphatic filtration of oil, given the small dispersion particles in the porous environment during the oil filtration process, is written as follows:

$$\begin{cases} \beta h(x)\dfrac{\partial P}{\partial t} = \dfrac{\partial}{\partial x}\left(\dfrac{k(x)\,h(x)}{\mu}\dfrac{\partial P}{\partial x}\right) - Q, \quad 0 < x < l \\ \dfrac{d\eta}{dt} = \lambda\left(\theta_0 - \gamma\eta\right), \quad 0 < t < T \end{cases} \tag{1}$$

The initial and boundary conditions are as follows

$$\begin{cases} P(x) = P_H, \;\; \eta(t) = \eta_0, \;\; t = 0, \;\; \dfrac{\partial P}{\partial x} = 0, \;\; x = 0, \;\; \dfrac{\partial P}{\partial x} = 0, \;\; x = L, \\ \oint\limits_{s_{i_q}} \dfrac{k(x)\,h(x)}{\mu}\dfrac{\partial P}{\partial x}dx = -q_{i_q}(t), \quad i_q = 1, N_q, \;\; Q = \sum\limits_{i_q=1}^{N_q} \delta_{i_q} q_{i_q} \\ m = m_1 + \eta\left(m_0 - m_1\right), \;\; k = k_0(1-\sqrt{\eta})^3, \;\; \beta = m\beta_H + \beta_c. \end{cases} \tag{2}$$

Applying the first equation for the discrete sphere and its boundary conditions, we come to the question of the three-diagonal constant and use the prognostic method to solve it. The velocity of the particles' squeezing in a porous medium $\eta$ (1) the second equation of the equation system $\tau$ we will have the following formula

$$\eta_i = \frac{\lambda\Delta\tau\theta_0 + \hat{\eta}_i}{1 + \gamma\lambda\Delta\tau}$$

here $\hat{\eta}_i$ is the value $\eta_i$ of the previous time layer, its initial value is deducted from the initial condition. The value $\eta_i$ found is used to calculate the porosity and shear permeability $k$ coefficients of each time layer. In general, it is necessary to consider the gel particles during the oil filtration process so that the calculation results of oil ponds are close to the actual result in the oil pond environment. Because, as time goes by, oil and lubricant coefficients become smaller. This, in turn, requires the calculation of spindle and conductivity coefficients at any time. As a result, the pressure in the oil atmosphere is slowed down over time. Developed mathematical model can also be used to calculate key aspects of the software's oil fields, to analyze, predict and design the process.

# ON THE CENTRAL LIMIT THEOREM FOR WEAKLY DEPENDENT RANDOM VARIABLES WITH VALUES IN $D[0,1]$

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Let $\{X_n(t), t \in [0,1], n \geqslant 1\}$ be a sequence of random variables with values in $D[0,1]$ (a space of all real-valued, right continuous and with left limits functions on $[0,1]$ which is endowed with the Skorohod topology). We say that $\{X_n(t), t \in [0,1], n \geqslant 1\}$ satisfies a central limit theorem if the distribution of $\frac{1}{\sqrt{n}}(X_1(t) + ... + X_n(t))$ converges weakly to a Gaussian distribution in $D[0,1]$. The central limit theorem for the sequence of independent identically distributed random variables with values in $D[0,1]$ were studied by many authors (see [1] and references therein).

As a measure of weak dependence we use the mixing coefficients. For a given sequence$\{X_n(t), t \in [0,1], n \geqslant 1\}$ of $D[0,1]-$ valued random variables mixing coefficients are defined as following:

$$\rho_m(n) = \sup_{R^m} \sup \left\{ \frac{|E(\xi - E\xi)(\eta - E\eta)|}{E^{\frac{1}{2}}(\xi - E\xi)^2(\eta - E\eta)^2} : \xi \in L_2(F_1^k), \eta \in L_2(F_{n+k}^{\infty}), k \in N \right\}.$$

where $F_a^b - \sigma$-field generated by $\prod_m X_a(t), ..., \prod_m X_b(t), \prod_m : D[0,1] \to R^m$-operator of projection from $D[0,1]$ to $R^m$, i.e. $\prod_m X(t) = \{X(t_1), ..., X_n(t_m)\}$ and $L_2(F_a^b)$-the space of square integrable and $F_a^b$ - measurable random variables.
Our main result is the following theorem.

**Theorem.** *Let $\{X_n(t), t \in [0,1], n \geqslant 1\}$ be a strictly stationary sequence of $\rho-$ mixing random variables with values in $D[0,1]$ such that $EX_1(t) = 0, E|X_1(t)|^{2+\varepsilon} < \infty$ for all $t \in [0,1]$ and some $\varepsilon > 0$. Assume that there exist nondecreasing continuous functions $F$ on $[0,1]$ such that for all $0 \leqslant s \leqslant t \leqslant 1$ the following hold:*

$$E|X_1(s) - X_1(t)|^{2+\varepsilon} \leqslant (F(s) - F(t)) \, log^{-(3+2\varepsilon)} \left(1 + (F(s) - F(t))^{-1}\right),$$

$$\sum_{k=1}^{n} \rho_m^{\frac{2}{2+\varepsilon}}(2^k) < \infty, \quad m = 1, 2, ...$$

*Then $\{X_n(t), t \in [0,1], n \geqslant 1\}$ satisfies the central limit theorem and the limiting mean-zero, sample continuous Gaussian process has covariance function:*

$$F(t_1, t_2) = \lim_{n\to\infty} \frac{1}{n} ES_n(t_1)S_n(t_2), t_1, t_2 \in [0,1].$$

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## ONE OPTIMAL INTERPOLATION FORMULAS WITH DERIVATIVES IN SOBOLEV SPACE

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We consider the following interpolation formula

$$\varphi(z) \cong \sum_{\beta=0}^{N} C_\beta(z)\varphi(h\beta) + \sum_{n=1}^{s} (A_n(\ z)\varphi^{(2n-1)}(0) + B_n(z)\varphi^{(2n-1)}\ (1)) \tag{1}$$

The following difference is called the error of the formula (1)

$$(\ell,\varphi) = \varphi(z) - \sum_{\beta=0}^{N} C_\beta(z)\varphi(h\beta) - \sum_{n=1}^{s} (A_n(\ z)\varphi^{(2n-1)}(0) + B_n(z)\varphi^{(2n-1)}\ (1)) \tag{2}$$

$\ell(x)$ is the error functional and has the form

$$\ell(x) = \delta(x-z) - \sum_{\beta=0}^{N} C_\beta(z)\delta(x-h\beta) + \sum_{n=1}^{s} (A_n(z)\delta^{(2n-1)}(x) + B_n(z)\delta^{(2n-1)}\ (x-1))$$

Here $C_\beta(z),\ \beta=\overline{0,N},\ A_n(z),\ B_n(z)$are coefficients of (1), $h=\frac{1}{N},\ \ N=1,2,...,\delta(x)$ is the Dirac delta function,$\varphi \in L_2^{(m)}(0,1),\ m=2s,\ s=1,2,3,...,\ L_2^{(m)}(0,1)$ is the Sobolev space of functions square integrable with m-th derivative .

For the functions of the space $L_2^{(m)}(0,1)$ the error (2) of the formula (1) is estimated by the norm of the error functional as follows

$$|(\ell,\varphi)| \le \|\varphi|L_2^{(m)}\| \cdot \|\ell|L_2^{(m)*})\|.$$

The formula (1) with coefficients $C_\beta(z),\ A_n(z), B_n(z)$which give the minimum to the norm of the error functional $\ell$ is called optimal interpolation formula in the space $L_2^{(m)}(0,1)$.

Thus in order to construct an optimal interpolation formula of the form (1) we should solve the following problems.

**Problem 1**. Find the norm $\|\ell|L_2^{(m)*})\|$ of the error functional$\ell$.

**Problem 2**. Find such coefficients $C_\beta(z),\ A_n(z), B_n(z)$ which attain the equality

$$\|\overset{\circ}{\ell}\| = \underset{C_\beta(z),\, A_n(z), B_n(z)}{inf} \|\ell\|.$$

In the present work Problem1 is solved and for the solution of Problem 2 the following system of linear equations is obtained

$$\sum_{\gamma=0}^{N} C_\gamma \frac{|h\beta-h\gamma|^{4s-1}}{2(4s-1)!} - \sum_{\alpha=1}^{s}\left(A_\alpha \frac{(h\beta)^{4s-2\alpha}}{2(4s-2\alpha)!} - B_\alpha \frac{(1-h\beta)^{4s-2\alpha}}{2(4s-2\alpha)!}\right) +$$

$$+\sum_{\alpha=0}^{2s-1} \lambda_\alpha (h\beta)^\alpha = \frac{|h\beta-h\gamma|^{4s-1}}{2(4s-1)!},\quad \beta=\overline{0,N}$$

$$\sum_{\gamma=0}^{N} C_\gamma \frac{(h\gamma)^{4s-2n}}{2(4s-2n)!} - \sum_{p=1}^{s} \frac{B_p}{2(4s-2p-2n+1)!} - (2n-1)!\lambda_{2n-1} = \frac{z^{4s-2n}}{2(4s-2n)!}, \quad n = \overline{1,s}$$

$$\sum_{\gamma=0}^{N} C_\gamma \frac{(1-h\gamma)^{4s-2n}}{2(4s-2n)!} - \sum_{p=1}^{s} \frac{A_p}{2(4s-2p-2n+1)!} + \sum_{\alpha=2n-1}^{2s-1} \frac{\alpha!\lambda_\alpha}{(\alpha-2n+1)!} = \frac{(1-z)^{4s-2n}}{2(4s-2n)!}, \quad n = \overline{1,s}$$

$$\sum_{\gamma=0}^{N} C_\gamma (h\gamma)^\alpha = \frac{\alpha!(1-(-1)^\alpha)}{2} A_{\frac{\alpha+1}{2}} - \sum_{p=1}^{\left[\frac{\alpha+1}{2}\right]} \frac{\alpha! B_p}{2(\alpha-2p+1)!} - (2n-1)!\lambda_{2n-1}, \quad \alpha = \overline{0, 2s-1}.$$


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## ASYMPTOTICAL PROPERTIES OF LIKELIHOOD RATIO STATISTICS BY INCOMPLETE OBSERVATIONS


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The likelihood ratio statistics plays an important role in decision theory. For example, while testing the simple hypothesis $H_0$ against complicated alternative $H_1$ about unknown law of distributions the criterions constructed on likelihood ratio statistics, according to Newmann-Pirson's lemma, are uniformly more powerful for any size n of observations (see [1-2]). Here appears some useful for estimation theory and hypothesis testing asymptotical properties of likelihood ratio statistics, when alternative $H_1$ depends on $n$ and close to $H_0$, i.e. $H_1 = H_{1n} \to H_0$ for $n \to \infty$. These properties are local and uniform asymptotic normality of likelihood ratio statistics. The essence of locally asymptotically normality that likelihood ratio statistics admits approximation by functions of the type $\exp\{u\omega_{n,\theta} - \frac{1}{2}u^2\}$, where $\omega_{n,\theta}$ is asymptotical normal (at $n \to \infty$) random variables with parameters (0,1). There are set of papers on investigations of the locally asymptotically normality for likelihood ratio statistics and it's applications in statistics. The most remarkable works are [2-4], which shown that local asymptotic normality allows the development of asymptotical theory for most maximum likelihood and Bayesian type estimators and proved the contiguality properties of the family of probability distributions. In papers [5-8] established properties of locally asymptotically normality for likelihood ratio statistics in competing risks model under random censoring of observations on the right and both sides. In this paper, we consider cases when the observed random variables create Markov chain. We investigate local asymptotic normality for likelihood ratio statistics in random censoring models.


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## LENGTH OF NATURALLY-GRADED 2-FILIFORM LIE ALGEBRAS

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The knowledge of the graded algebras of a particular Lie algebra class gives a valuable information about the structure of that class. The classes of filiform and quasi-filiform algebras are very important subclass of nilpotent algebras. The gradations of algebras are very important to investigation of the structural theory and properties of those algebras.The gradation of algebra is one of the useful aspect to investigation of structure of nilpotent Lie algebras.

Naturally graded algebras are very important subclass of nilpotent Lie algebras. It is well-known that the description of nilpotent Lie algebras can be obtained using the classification of naturally graded nilpotent algebras [1]. The maximal gradation is useful to the description of derivations and cohomological groups of algebra [3]. Since the maximal gradation is a graduation of length n, the next step on this considiration is investigation algebras of length $n-1$. In this work we describe some 2-filiform Lie algebras of length $n-1$.

**Definition 1.** An algebra $(L, [-,-])$ over a field $F$ is called Lie algebra if for any $x, y, z \in L$ the following identities:

$$[x[y,z]] + [y[z,x]] + [z[x,y]]$$

$$[x,x] = 0$$

holds.

For an arbitrary Lie algebra $L$ we define the lower central series:

$$L^1 = L,\ L^{k+1} = [L^k, L],\ k \geq 1.$$

**Definition 2.** An $n$-dimensional Lie algebra $L$ is called nilpotent if there exist $s \in N$ such that $L^s = 0$. The minimal number of this property is called nilindex of the algebra $L$.

An $n$-dimensional Lie algebra $L$ is called quasi-filiform if $L^{n-2} \neq 0$ and $L^{n-1} = 0$, i.e., an algebra with nilindex $n-1$.

Let $L$ be a $Z$-graded Lie algebra with a finite number of non zero subspaces, i.e. $L = \bigoplus_{i \in Z} V_i$, where $[V_i, V_j] \subseteq V_{i+j}$ for any $i, j \in Z$. We say that a Lie algebra $L$ admits a *connected gradation* if $L = V_{k_1} \oplus V_{k_2} \oplus \cdots \oplus V_{k_t}$, where each $V_i$ is non-zero for $k_1 \leq i \leq k_t$.

The number of subspaces $l(\oplus L) = k_t - k_1 + 1$ is called the length of the gradation. The length $l(L)$ of a Lie algebra $L$ is defined as

$$l(L) = \max\{l(\oplus L) = k_t - k_1 + 1 \mid L = V_{k_1} \oplus V_{k_2} \oplus \cdots \oplus V_{k_t} \quad \text{is a connected gradation.}\}$$

Now we define the natural gradation for the nilpotent Lie algebra $L$.

**Definition 3.** For a given nilpotent Lie algebra $L$ with nilindex $s$, put $L_i = L^i/L^{i+1}$, $1 \leq i \leq s$ and $gr(L) = L_1 \oplus L_2 \oplus L_3 \oplus \ldots \oplus L_s$. Then $[L_i, L_j] \subseteq L_{i+j}$ and we obtain the graded algebra $gr(L)$. If $gr(L)$ and $L$ are isomorphic, then we say that an algebra $L$ is naturally graded.

Note that there are three naturally graded quasi-filiform Lie algebras.

**Theorem 1** [2]. *Let $L$ be a naturally graded quasi-filiform Lie algebra. Then it is isomorphic to one of the following pairwise non-isomorphic algebras*

$$Q_(n,r)(n \geq 7, n \quad odd, r \quad odd, 3 \leq r \leq n-4: \begin{cases} [e_0, e_i] = e_{i+1}, 1 \leq i \leq n-3, \\ [e_i, e_{r-i}] = (-1)^{i-1} e_{n-1}, 1 \leq i \leq \frac{r-1}{2} \\ [e_i, e_{n-2-i}] = (-1)^{i-1} e_{n-2}, 1 \leq i \leq \frac{n-3}{2} \end{cases}$$

$$T_(n, n-3)(n \quad even, n \geq 6): \begin{cases} [e_o, e_i] = e_{i+1}, 1 \leq i \leq n-3, \\ [e_{n-1}, e_1] = \frac{n-4}{2} e_{n-2}, \\ [e_i, e_{n-3-i}] = (-1)^{i-1}(e_{n-3} + e_{n-1}) 1 \leq i \leq \frac{n-4}{2}, \\ [e_i, e_{n-2-i}] = (-1)^{i-1} \frac{n-2-2i}{2} e_{n-2} 1 \leq i \leq \frac{n-4}{2}, \end{cases}$$

$$T_(n, n-4)(n \quad odd, n \geq 7): \begin{cases} [e_o, e_i] = e_{i+1}, 1 \leq i \leq n-3, \\ [e_{n-1}, e_i] = \frac{n-5}{2} e_{n-4+i}, 1 \geq i \geq 2, \\ [e_i, e_{n-4-i}] = (-1)^{i-1}(e_{n-4} + e_{n-1}) 1 \leq i \leq \frac{n-5}{2}, \\ [e_i, e_{n-3-i}] = (-1)^{i-1} \frac{n-3-2i}{2} e_{n-3} 1 \leq i \leq \frac{n-5}{2}, \\ [e_i, e_{n-2-i}] = (-1)^{i} \frac{n-3-i}{2} e_{n-2} 2 \leq i \leq \frac{n-3}{2}, \end{cases}$$

In the following Proposition we find the length of the naturally graded quasi-filiform Lie algebras, which given in Theorem 1.

**Proposition.** *The length of quasi-filiform Lie algebra are follows:*

$$l(Q_{(n,r)}) = n-1, \quad l(T_{(n,n-3)}) = n-1, \quad l(T_{(n,n-4)}) = n-1.$$

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## ON LOCAL LOGARITHMIC RESIDUE

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Let $D \subset \mathrm{C}^n$ be a bounded domain, consider the set of zeros of the map

$$f = (f_1, ..., f_n) : E_f = \{a \in D : f_1(a) = ... = f_n(a) = 0\}.$$

Suppose $E_f$ is discrete in $D$. Denote by $\omega(f)$ the form

$$\omega(f) = \frac{(n-1)!}{(2\pi i)^n} \sum_{k=1}^{n} (-1)^{k-1} \overline{f_k}\, |f|^{-2n}\, d\overline{f}[k] \wedge df,$$

where as usual $df = df_1 \wedge ... \wedge df_n$, $|f|^2 = |f_1|^2 + ... + |f_n|^2$, and the $\mu_a$ is multiplicity of zero.

The classical formula for a multidimensional logarithmic residue is formulated as follows:

**Theorem** (Yuzhakov-Rus, [1], [2]). *Let $f = (f_1, ..., f_n)$ be a holomorphic mapping defined in some neighborhood of the closure of a bounded domain $D \subset \mathrm{C}^n$ with a piecewise smooth boundary, $f$ does not have zeros on $\partial D$, but $\varphi \in \mathcal{O}(D) \cap (\overline{D})$, then*

$$\int_{\partial D} \varphi(\zeta)\omega(f) = \sum_{a \in E_f \cap D} \mu_a \varphi(a).$$

Now let $\varphi \in C^\infty(D)$. Then the following statement is true.

**Theorem.** *Let $f = (f_1, ..., f_n)$ be a holomorphic mapping defined in some neighborhood of the closure of a bounded domain $D \subset \mathrm{C}^n$ with a piecewise smooth boundary, $f$ does not have zeroes on $\partial D$, but $\varphi \in C^\infty(\overline{D})$, then*

$$\int_{\partial D} \varphi\omega(f) = \sum_{a \in E_f} \mu_a \varphi(a) - \int_{D} \omega(f) \wedge \overline{\partial}\varphi,$$

*where operator is* $\overline{\partial} = \sum_{k=1}^{n} d\overline{\zeta_k} \frac{\partial}{\partial \overline{\zeta_k}}$.


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## ON PROPERTIES OF INJECTIVITY AND NUCLEARITY FOR REAL $C^*$-ALGEBRAS


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Let $B(H)$ be the algebra of all bounded linear operators, acting on a complex Hilbert space $H$. Recall that a weakly closed *-subalgebra $R \subset B(H)$ with the identity $\mathbf{1}$ is called a W*-*algebra*. A real *-subalgebra $R \subset B(H)$ with $\mathbf{1}$ is called a *real* W*-*algebra*, if it is weakly closed and $R \cap iR = \{0\}$. A Banach *-algebra $A$ is called a C*-*algebra*, if $\|aa^*\| = \|a\|^2$ for any $a \in A$. A real Banach *-algebra $R$ is called a *real* C*-*algebra*, if $\|aa^*\| = \|a\|^2$ and the element $\mathbf{1} + aa^*$ is invertible for any $a \in R$. It is easy to see that $R$ is a real C*-algebra if and only if a norm on $R$ can be extended onto the complexification $A = R + iR$ of the algebra $R$ so that algebra $A$ is a C*-algebra (see 5.1.1 in [1]).

We say that a C*-algebra $R$ is called *injective* if the following condition is held: for every C*-algebra $Q$, for every selfadjoint linear subspace $S$ of $Q$, containing the identity $\mathbf{1}$, and for every completely positive linear map $\varphi : S \to R$, there is a completely positive linear map $\overline{\varphi} : Q \to R$ such that $\overline{\varphi}|_S = \varphi$. Recall that a real W*-algebra $R \subset B(H)$ is injective if and only if it has the *property of E* (the extension property), i.e. there exists a projection $P : B(H) \to R$ such that $\|P\| = 1$, $P(\mathbf{1}) = \mathbf{1}$.(see [2]).

A C*-algebra $R$ is called *nuclear*, if for any C*-algebra $Q$, there is the unique C*-norm on the algebraic tensor product $R \otimes Q$.

**Lemma 1.** *Let $R$ be a nuclear real C*-algebra. Then, its commutant $R'$ is an injective real W*-algebra.*

**Theorem 1.** *A real C*-algebra $R$ is nuclear if and only if the real W*-algebra $R^{**}$ is injective, where $R^{**}$ is the second dual space of $R$.*

**Theorem 2.** *A real C*-algebra $R$ is nuclear if and only if its enveloping C*-algebra $R + iR$ is nuclear.*

From theorems 1 and 2, we have the following:

**Corollary 1.** *The properties of injectivity and nuclearity are equivalent for real W*-algebras.*

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## ON THE PERTURBATION OF LINEAR EQUATIONS IN THE CASE OF AN INCOMPLETE GENERALIZED JORDAN SET


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In this article by reduction methods developed in works [1,2], the problem of perturbation of an linear equations in the case of an incomplete generalized Jordan set is considered.

Let $E_1, E_2$ be some Banach spaces, a linear operator-function $A(\varepsilon) \in L(E_1, E_2)$ be analytically dependent on a small parameter $\varepsilon \in \mathbb{C}$ , moreover $A(0) = B$ be a Fredholm operator with $Ker(B) = \{\varphi_1^{(1)}, \ldots, \varphi_n^{(1)}\}$, $Ker(B^*) = \{\psi_1^{(1)}, \ldots, \psi_n^{(1)}\}$, and incomplete Generalized Jordan set (GJS) $\{\varphi_i^{(s)}\}_{i=\overline{1,n}}^{s=\overline{1,p_i}}$

$$B\varphi_i^{(s)} = \sum_{k=1}^{s-1} A_k \varphi_i^{(s-k)}, \ \left\langle \sum_{k=1}^{s-1} A_k \varphi_i^{(s-k)}, \psi_j^{(1)} \right\rangle = 0, \ s = \overline{2, p_i}, i, j = \overline{1, n}, \tag{1}$$

$$\left\langle \sum_{k=1}^{p_i} A_k \varphi_i^{(p_i+1-k)}, \psi_j^{(1)} \right\rangle \neq 0, \ i, j = \overline{1, n}, \ D_p = det \left\| \left\langle \sum_{k=1}^{p_i} A_k \varphi_i^{(p_i+1-k)}, \psi_j^{(1)} \right\rangle \right\| = 0.$$

Let $\{\gamma_i\}_{i=\overline{1,n}}$, $\{z_i\}_{i=\overline{1,n}}$ be biorthogonal systems to $\{\varphi_i^{(1)}\}_{i=\overline{1,n}}$, $\{\psi_i^{(1)}\}_{i=\overline{1,n}}$ respectively.

For every $i = \overline{1, n}$ we introduce the operators $B_i = B + \sum\limits_{j \neq i} \langle \cdot, \gamma_j \rangle z_j$. It's easy to make sure that $N(B_i) = \{\varphi_i^{(1)}\}$, $N^*(B_i) = \{\psi_i^{(1)}\}$. Consider the perturbation operator-functions $\overline{A}_i(\varepsilon) = B_i - \sum\limits_{k=1}^{\infty} \varepsilon^k A_k$.

The following theorems are true.

**Theorem 1.** *In order for all the chains of the GJS* $\{\varphi_i^{(j)}\}_{i=\overline{1,n}}^{j=\overline{1,p_i}}$ *it is necessary and sufficient that there is a number* $\rho_0$ *such that for all* $\varepsilon$, *satisfying the inequality* $0 < |\varepsilon| \leq \rho_o$, *the operators* $\overline{A}_i^{-1}(\varepsilon)$, $i = \overline{1, n}$, *exist and are limited.*

**Theorem 2.** *Let* $B \in L(E_1, E_2)$ *be a Fredholm operator with number of zeros* $n > 1$ *which has an incomplete* $A-$ *GJChs of finite length* $p_i, i = \overline{1, n}$. *Then each equation*

$B_i = h + \sum\limits_{k=1}^{\infty} \varepsilon^k A_k y$ *has a unique solution* $y_i(\varepsilon)$, *which, subject to* $p_i - q_i \leq 0$ *will be analytic at the point* $\varepsilon = 0$ *and in its some neighborhood, and provided* $p_i > q_i$, *they have at the point* $\varepsilon = 0$ *pole of order* $p_i - q_i$.


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## CRITICAL CURVE OF THE CROSS DIFFUSION SYSTEM COUPLED VIA NONLINEAR BOUNDARY FLUX


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In this thesis, we investigate the existence and non-existence of global weak solutions to the following cross diffusion system

$$\frac{\partial u}{\partial t} = \frac{\partial}{\partial x}\left(\upsilon^{m_1-1}\left|\frac{\partial u}{\partial x}\right|^{p-2}\frac{\partial u}{\partial x}\right), \quad \frac{\partial \upsilon}{\partial t} = \frac{\partial}{\partial x}\left(u^{m_2-1}\left|\frac{\partial \upsilon}{\partial x}\right|^{p-2}\frac{\partial \upsilon}{\partial x}\right), \quad x \in R_+, \ t > 0, \qquad (1)$$

$$-\upsilon^{m_1-1}\left|\frac{\partial u}{\partial x}\right|^{p-2}\frac{\partial u}{\partial x}(0,t) = u^{q_1}(0,t), \quad -u^{m_2-1}\left|\frac{\partial \upsilon}{\partial x}\right|^{p-2}\frac{\partial \upsilon}{\partial x}(0,t) = \upsilon^{q_2}(0,t), \ t > 0, \qquad (2)$$

$$u(x,0) = u_0(x), \quad \upsilon(x,0) = \upsilon_0(x), \quad x \in R_+, \qquad (3)$$

where $m_1$, $m_2 > 1$, $p > 2$, $q_1$, $q_2 > 0$, $u_0$, $\upsilon_0$ are nonnegative continuous functions with compact supports in $R_+$.

Cross-diffusion means that the spatial displacement of one object, described by one of the variables, occurs due to diffusion of another object described by another variable [1]. In particular, the simplest example is the parasite (the first object) that is inside the "master"(the second object), moves due to diffusion of the host. Cross-diffusion systems play a major role in the mathematical modeling of pigmentation of animals, localization of leukocytes moving in response to bacterial inflammation [2].

The main purposes of this paper are to obtain the conditions global existence of solutions and the critical global solvability exponent of system (1)-(3).

**Theorem 1.** *Let* $q_1 > (m_2+1)/p$, $q_2 > (m_1+1)/p$, *then each solution of problem (1)-(3) is unbounded at sufficiently large initial data.*

**Remark.** Theorem 1 shows that the critical global existence curve are $q_{1_0} = (m_2+1)/p$, $q_{2_0} = (m_1+1)/p$.


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## CRITERION FOR THE CLOSABILITY AND CLOSEDNESS OF TRIDIAGONAL OPERATOR MATRICES

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Let $n \in \mathbb{N}$, $n \geq 2$, let $\big(\mathcal{H}_i, \|\cdot\|_i\big)$, $i = 1, \cdots, n$, be Banach spaces, and let $\big(\mathcal{H}, \|\cdot\|\big)$ be the Euclidean product of $\mathcal{H}_1, \ldots, \mathcal{H}_n$, that is,

$$\mathcal{H} := \mathcal{H}_1 \oplus \cdots \oplus \mathcal{H}_n, \quad \|f\| := \sqrt{\|f_1\|_1^2 + \cdots + \|f_n\|_n^2}, \quad f = (f_1, \ldots, f_n)^t \in \mathcal{H}.$$

In the Banach space $\mathcal{H}$ we consider linear operators $\mathcal{A}$ that admit an $n \times n$ block operator matrix representation,

$$\mathcal{A} = (A_{ij})_{i,j=1}^n \ \text{ in } \mathcal{H} = \mathcal{H}_1 \oplus \cdots \oplus \mathcal{H}_n,$$

where the entries $A_{ij} : \mathcal{H}_j \supset D(A_{ij}) \to \mathcal{H}_i$, $i, j = 1, \ldots, n$, are densely defined closable linear operators and for which the domain of $\mathcal{A}$, given by

$$D(\mathcal{A}) = \bigoplus_{j=1}^{n} \Big( \bigcap_{i=1}^{n} D(A_{ij}) \Big),$$

is again dense in $\mathcal{H}$.

The talk is based on the joint result with C.Tretter [1]. In the following Theorem we derive a criterion for the closability and closedness, respectively, of *tridiagonal* block operator matrices which are characterized by $A_{ij} = 0$ for $|i - j| > 1$ and it generalizes and improves, the result for $n = 2$ in [2].

**Theorem 1.** *Let $\mathcal{A}$ be tridiagonal and diagonally dominant and $\delta_{ij}$ be the $A_{ii}$-bound of $A_{ij}$, $i, j = 1, ..., n$, $|i - j| = 1$. If*

$$\delta_n(\mathcal{A}) = \cfrac{\delta_{n,n-1}\delta_{n-1,n}}{1 - \cfrac{\delta_{n-1,n-2}\delta_{n-2,n-1}}{1 - \cfrac{\delta_{n-2,n-3}\delta_{n-3,n-2}}{1 - \cfrac{\ddots}{\ddots 1 - \cfrac{\delta_{32}\delta_{23}}{1 - \delta_{21}\delta_{12}}}}}} < 1, \qquad (1)$$

*then $\mathcal{A}$ is closable and closed if its diagonal elements $A_{ii}$ are closed.*

Remark. The continued fraction $\delta_n(\mathcal{A})$ in (1) can also be defined by the recursion

$$\delta_1(\mathcal{A}) := 0, \quad \delta_k(\mathcal{A}) := \frac{\delta_{k,k-1}\delta_{k-1,k}}{1-\delta_{k-1}(\mathcal{A})}, \quad k = 2, 3, \dots, n.$$

Then, the condition (1) is equivalent to $\delta_{k,k-1}\delta_{k-1,k} + \delta_{k-1}(\mathcal{A}) < 1$, $k = 2, 3, \dots, n$.

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## NUMERICAL RANGE OF A FRIEDRICH'S MODEL WITH RANK TWO PERTURBATION


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Let $\mathcal{H}$ be a complex Hilbert space and let $\mathcal{A}$ be a bounded linear operator in $\mathcal{H}$. Then the numerical range of $\mathcal{A}$ is the set

$$W(\mathcal{A}) := \{(\mathcal{A}f, f) : f \in \mathcal{H}, \ \|f\| = 1\}.$$

By the well-known Toeplitz-Hausdorff theorem, the numerical range is a convex subset of $\mathbb{C}$ and it satisfies the so-called *spectral inclusion property*

$$\sigma_{\mathrm{p}}(\mathcal{A}) \subset W(\mathcal{A}), \quad \sigma(\mathcal{A}) \subset \overline{W(\mathcal{A})}$$

for the point spectrum $\sigma_{\mathrm{p}}(\mathcal{A})$ (or set of eigenvalues) of $\mathcal{A}$ and the spectrum $\sigma(\mathcal{A})$ of $\mathcal{A}$, see [1]. In this note we investigate the numerical range of a Friedrichs model with rank two perturbation. We establish that under some conditions its numerical range and spectrum are coincide.

Let $L_2(\mathbb{T}^3)$ be the Hilbert space of square-integrable (complex) functions defined on be the three-dimensional torus $\mathbb{T}^3$.

Let us consider an operator $H$ acting on the Hilbert space $L_2(\mathbb{T}^3)$ as $H := H_0 - V_1 + V_2$, where $(H_0 f)(p) = u(p)f(p)$, $f \in L_2(\mathbb{T}^3)$, and $V_\alpha$, $\alpha = 1, 2$ are interaction operators,

$$(V_\alpha f)(p) = \mu_\alpha v_\alpha(p) \int_{\mathbb{T}^3} v_\alpha(t) f(t) dt, \quad f \in L_2(\mathbb{T}^3).$$

Here $\mu_\alpha$, $\alpha = 1, 2$, are positive reals, the function $u(\cdot)$ is a real-valued analytic function on $\mathbb{T}^3$, it has an unique non-degenerate minimum at $p_1 \in \mathbb{T}^3$ and an unique non-degenerate maximum at $p_2 \in \mathbb{T}^3$. The functions $v_\alpha(\cdot)$, $\alpha = 1, 2$ are real-valued on $\mathbb{T}^3$, having partial derivatives up to the third order inclusive on some neighborhood of $p_\alpha \in \mathbb{T}^3$. We assume also that $\mathrm{mes}(\mathrm{supp}\{v_1(\cdot)\} \cap \mathrm{supp}\{v_2(\cdot)\}) = 0$.

Set $E_1 := \min\limits_{p \in \mathbb{T}^3} u(p)$, $E_2 := \max\limits_{p \in \mathbb{T}^3} u(p)$ and

$$\mu_\alpha^0 := (-1)^{\alpha+1} \left( \int_{\mathbb{T}^3} \frac{v_\alpha^2(t)dt}{u(t) - E_\alpha} \right)^{-1}, \quad \alpha = 1, 2.$$

The main result of this note is the following

**Theorem.** a) *If $\mu_\alpha = \mu_\alpha^0$ and $v_\alpha(p_\alpha) = 0$ for $\alpha = 1, 2$, then $W(H) = \sigma(H) = [E_1; E_2]$;*
b) *If $\mu_\alpha = \mu_\alpha^0$ and $v_\alpha(p_\alpha) \neq 0$ for $\alpha = 1, 2$, then $W(H) = (E_1; E_2)$;*
c) *If $\mu_\alpha = \mu_\alpha^0$ and $v_\alpha(p_1) = 0$, $v_\alpha(p_2) \neq 0$, then $W(H) = [E_1; E_2)$;*
d) *If $\mu_\alpha = \mu_\alpha^0$ and $v_\alpha(p_1) \neq 0$, $v_\alpha(p_2) = 0$, then $W(H) = (E_1; E_2]$.*

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## ON THE NUMBER OF EIGENVALUES OF A TWO-CHANNEL MOLECULAR RESONANCE MODEL


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For a fixed $h > 0$ we denote by $\mathbb{T}_h^{\mathrm{d}}$ the $d$-dimensional cube $(-\pi/h; \pi/h]^{\mathrm{d}}$ with appropriately identified sides. Let $\mathcal{H}_0 := \mathbb{C}$ be the field of complex numbers (channel 1) and $\mathcal{H}_1 := L_2(\mathbb{T}_h^{\mathrm{d}})$ be the Hilbert space of square-integrable (complex) functions defined on $\mathbb{T}_h^{\mathrm{d}}$ (channel 2).

In this note we consider an operator matrix $\mathcal{A}$ acting on the two-channel Hilbert space $\mathcal{H} := \mathcal{H}_0 \oplus \mathcal{H}_1$ as

$$\mathcal{A}_h := \begin{pmatrix} A_{00}(h) & A_{01}(h) \\ A_{01}(h)^* & A_{11}(h) \end{pmatrix},$$

with elements $A_{ij}(h) : \mathcal{H}_j \to \mathcal{H}_i$, $i = 0, 1$, $i \leq j$:

$$A_{00}(h) f_0 = -\varepsilon f_0, \quad A_{01}(h) f_1 = \alpha \int_{\mathbb{T}_h^{\mathrm{d}}} v(t) f_1(t) dt,$$

$$(A_{11}(h) f_1)(x) = (-\varepsilon + w(x)) f_1(x) - \beta \varphi(x) \int_{\mathbb{T}_h^{\mathrm{d}}} \varphi(t) f_1(t) dt.$$

Here $\varepsilon$, $\alpha$, $\beta$ are positive reals, the functions $w(\cdot)$, $v(\cdot)$, $\varphi(\cdot)$ are real-valued continuous functions on $\mathbb{T}_h^{\mathrm{d}}$. Under these assumptions the operator matrix $\mathcal{A}_h$ (so-called a two-channel molecular resonance model) is bounded and self-adjoint.

We remark that operator matrix $\mathcal{A}_h$ is associated with Hamiltonian of a system consisting of at most two particles on a d-dimensional non-integer lattice $(hZ)^{\mathrm{d}}$, interacting via both a pair contact potential ($\beta > 0$) and creation and annihilation operators ($\alpha > 0$).

It is clear that $\sigma_{\text{ess}}(\mathcal{A}_h) = [-\varepsilon + m_h; -\varepsilon + M_h]$, where

$$m_h := \min_{x \in \mathbb{T}_h^{\mathrm{d}}} w(p), \quad M_h := \max_{x \in \mathbb{T}_h^{\mathrm{d}}} w(p).$$

Assume that $w(\cdot)$ is a non-negative function on $\mathbb{T}_h^{\mathrm{d}}$ and set

$$\beta_0(h) := \left( \int_{\mathbb{T}_h^{\mathrm{d}}} \frac{\varphi^2(t)dt}{w(t) - m_h} \right)^{-1} \quad \text{with} \quad \int_{\mathbb{T}_h^{\mathrm{d}}} \frac{\varphi^2(t)dt}{w(t) - m_h} < \infty.$$

The main result of this note is the following

**Theorem.** *Let $h > 0$ be a fixed. For any $\alpha, \beta > 0$ the operator $\mathcal{A}_h$ has no more than two (resp. one) eigenvalues lying on the l.h.s. of $m_h$ resp. on the r.h.s. of $M_h$. If $\beta \in (0; \beta_0(h))$, then for any $\alpha > 0$ the operator $\mathcal{A}_h$ has an unique eigenvalue lying on the l.h.s. of $m_h$.*

An existence and analyticity of eigenvalues of a two-channel molecular model are studied in [1] for the case $h = 1$.

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# DISCRETE-TIME CHAOTIC DYNAMICS OF LESLIE'S PREY-PREDATOR MODEL


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In [3] Leslie's prey-predator model in continuous time is considered.

We consider (as in [2]) the discrete time Leslie's prey-predator model, which has the following form

$$V : \begin{cases} x^{(1)} = x(a - 1 - bx - cy) \\ y^{(1)} = y(d - 1 - \alpha \frac{y}{x}), \end{cases} \tag{1}$$

where $a > 0, b > 0, c > 0, d > 0, \alpha > 0$. We consider this operator on $R_+^2 = \{(x, y) \in R^2 : x > 0, y \geq 0\}$.

The set $M$ is called an *invariant* with respect to operator $V$ if $V(M) \subset M$.

**Theorem 1.** *The following sets are invariant with respect to operator $V$:*

1. $M_1 = \{(x, y) \in R_+^2 | y = 0\}$;

2. *If* $1 < d \leq 2, 1 < a \leq 2$ *then*

$$M_2 = \left\{ (x, y) \in R_+^2 | 0 < x < \frac{\alpha(a-1)}{b\alpha + cd - c}, \quad 0 \leq y \leq \frac{(d-1)x}{\alpha} \right\}.$$

The restriction of the system () on $M_1$ is topologically conjugate to the famous quadratic family $F_\mu(x) = \mu x(1-x)$ (see [1]). Let $f: A \to A$ and $g: B \to B$ be two maps. $f$ and $g$ are called *topologically conjugate*([1]) if there exists a homeomorphism $h: A \to B$ such that, $h \circ f = g \circ h$. In the operator $V$, if $y = 0$ then $x^{(1)} = f_{a,b}(x) = x(a - 1 - bx)$.

**Theorem 2.** *Two maps* $F_\mu(x)$ *and* $f_{a,b}(x)$ *are topologically conjugate for* $a = 3 - \mu$.

The following theorem gives the limit points of the system on invariants $M_i$, $i = 1, 2$.

**Theorem 3.** *Let* $1 < d \leq 2$. *Then*

(i) If $4 < a < 2 + \sqrt{6}$ then the operator $V$ has 2-periodic orbits.

(ii) If $2 + \sqrt{6} < a < 4.544$ then the operator $V$ has 4-periodic orbits.

(iii) If $a > 3 + \sqrt{5}$ then the operator $V$ has chaotic dynamics (see [1] for definitions).

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## CIRCLE HOMEOMORPHISMS WITH TWO CRITICAL POINTS OF DIFFERENT ORBITS


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Let $f$ be an orientation preserving homeomorphism of the circle $S^1 \equiv \mathbb{R}^1/\mathbb{Z}^1$ with lift $F: \mathbb{R}^1 \to \mathbb{R}^1$, which is continuous, strictly increasing and fulfills $F(x+1) = F(x)+1, \forall x \in \mathbb{R}$. The circle homeomorphism $f$ is then defined by $f(x) = F(x)\ (mod\,1)$, $x \in S^1$. An important characteristic of circle homeomorphism is **rotation number** defined by Poincaré [1] as

$$\rho_f = \lim_{n \to \infty} \frac{F^n(x)}{n} (mod\,1).$$

The purpose of this paper is to study certain rigidity questions concerning critical circle mappings.

**Definition 1.** The point $x_{cr} \in S^1$ is called non-flat critical point of a homeomorphism $f$ with order $(2m+1)$, $m \in N$, if for a some $\delta-$ neighborhood $U_\delta(x_{cr})$, the function $f$ belongs to the class of $C^{2m+1}(U_\delta(x_{cr}))$ and

$$f'(x_{cr}) = f''(x_{cr}) = ... = f^{(2m)}(x_{cr}) = 0, \ f^{(2m+1)}(x_{cr}) \neq 0.$$

The order of the critical point $x_{cr}$ is $2m+1$. By a *critical circle map* we define an orientation preserving circle homeomorphism with exactly one non-flat critical point of odd type.

Yoccoz's classical theorem [2] states, that an analytic critical circle homeomorphism $f$ with irrational rotation number $\rho = \rho_f$ is topologically conjugate to a rigid rotation $f_\rho$ that is, there exists a homeomorphism $\varphi$ of the circle with $f = \varphi^{-1} \circ f_\rho \circ \varphi$. Graczyk and Swiatek in [3] proved that if $f$ is $C^3$ smooth circle homeomorphism with finitely many critical points of polynomial type and an irrational rotation number of bounded type, then the conjugating map $\varphi$ is singular function on $S^1$ i.e. $\varphi'(x) = 0$ a.e. on $S^1$. Consequently, the invariant measure of critical circle homeomorphisms is singular w.r.t. Lebesque measure on $S^1$.

Here then arises naturally the problem of regularity of the conjugasy between two critical circle maps with identical irrational rotation number. The regularity of conjugacy between two critical circle homeomorphisms with identical irrational rotation number studied in detail by E. de Faria and W. de Melo [4], K. Khanin and A. Teplinskii [5], A. Avila [6], D. Khmelev and M. Yampolskii [7]. Our main result is the following

**Theorem 1.** *Let $f_i \in C^3(S^1)$, $i = 1, 2$ be circle homeomorphisms with two critical points $x_1^{(i)}, x_2^{(i)}$ of the orders $2m_1^{(i)} + 1$ and $2m_2^{(i)} + 1$, respectively. Suppose that*

*(a) their rotation numbers $\rho_{f_i}$, $i = 1, 2$ are irrational and coincide i.e. $\rho_{f_1} = \rho_{f_2} = \rho$;*

*(b) the product of the order of critical points of $f_1$ and $f_2$ do not coincide i.e.*

$$(2m_1^{(1)} + 1) \cdot (2m_2^{(1)} + 1) \neq (2m_1^{(2)} + 1) \cdot (2m_2^{(2)} + 1);$$

*Then the map $h$ conjugating between $f_1$ and $f_2$ is a singular function.*

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## PURSUIT PROBLEM FOR THE NONLINEAR DIFFERENTIAL GAMES

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In this report we consider the Pursuit Problem, when objects move in dynamic flow field [1]. The proposed method substantiates the parallel approach strategy, i.e., the Π-strategy [2] for the nonlinear differential games. The new sufficient solvability conditions are obtained for problem of the pursuit.

Consider the differential game when Pursuer $X$ and Evader $Y$ having radius vectors $x$ and $y$ correspondingly move in the $\mathbb{R}^n$ and their dynamics will be described by the equations:

$$\dot{x} = u + f(t, x), \quad x(0) = x_0, \tag{1}$$

$$\dot{y} = v + f(t, y), \quad y(0) = y_0, \tag{2}$$

where $x, y \in \mathbb{R}^n, n \geq 2$; $x_0, y_0$ are the initial positions of the objects $X$ and $Y$. Let for $f(t,x)$ and $f(t,y)$ Caratheodory's conditions are satisfied on each compact set $Q$ of $R_+ \times \mathbb{R}^n$ and $|f(t,x) - f(t,y)| \leq k(t)|x-y|$ on $R_+ \times \mathbb{R}^n$. In (1) $u$ is control function of the Pursuer $X$ and the temporal variation of $u$ must be a measurable function $u(\cdot) : R_+ \to \mathbb{R}^n$ such that $|u(t)| \leq \alpha,$ for almost every $t \geq 0$. Similarly, in (2) $v$ is control function of the Evader $Y$ and the temporal variation of $v$ must be a measurable function $v(\cdot) : R_+ \to \mathbb{R}^n$ such that $|v(t)| \leq \beta,$ for almost every $t \geq 0$. In the (1)-(2) the objective of the Pursuer $X$ is to catch the Evader $Y$, i.e., reach the equality $x(t) = y(t)$ where $x(t)$ and $y(t)$ are trajectories generated during the game.

**Assumption 1.** *Let a) there exists positive root of the equation $\Psi(t) = 0$ with respect to $t$, where $\Psi(t) = |z_0| - (\alpha - \beta) \int_0^t \exp\left(-\int_0^s k(\tau)d\tau\right) ds$; b) satisfies the following condition $\alpha > \beta + k(t)\max_{t \in [0,T]} \Phi(t)$, where $T = \min\{t : \Psi(t) = 0\}$ and $\Phi(t) = \Psi(t) \exp \int_0^t k(s)ds$.*

**Theorem 1.** *Let Assumption 1 is satisfied. Then the Pursuer $X$ to catch the Evader $Y$ on the interval $[0, T]$ in the game (1)-(2), where $T$ is the smallest positive root of the equation $\Psi(t) = 0$.*

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# ON CONTINUATION OF THE SOLUTION OF A QUATERNIONIC DIRAC EQUATION

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Let $\Omega$ is a bounded simply connected domain in $R^3$ with boundary $\partial\Omega$ composed of a compact connected part $T$ of the plane $y_3 = 0$ and a smooth Lyapunov surface $S$ lying in the half-space $y_3 > 0$, with $\bar{\Omega} = \Omega \cup \partial\Omega$, $\partial\Omega = S \cup T$. As to $S$, we assume that each ray issuing from any point $x$ of the domain $\Omega$ intersects this surface at most $l$ points. We consider the the following Dirac equation (see [1]) for a free massive particle of spin 1/2:

$$D[\Psi] := (\gamma_0\partial_t - \sum_{k=1}^{3} \gamma_k\partial_k + im)[\Psi] = 0, \tag{1}$$

where the Dirac matrices [2, p.64] have the standard Dirac-Pauli form

$$\gamma_0 := \begin{Vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{Vmatrix}, \gamma_1 := \begin{Vmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{Vmatrix},$$

$$\gamma_2 := \begin{Vmatrix} 0 & 0 & 0 & i \\ 0 & 0 & -i & 0 \\ 0 & -i & 0 & 0 \\ i & 0 & 0 & -1 \end{Vmatrix}, \gamma_3 := \begin{Vmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{Vmatrix}, \tag{2}$$

and where $\partial_t := \frac{\partial}{\partial t}; \partial_k := \frac{\partial}{\partial x_k}, m \in R, \Psi : R^4 \to C^4$. Suppose that the spinor field $\Psi$ is time-harmonic (=monohromatic):

$$\Psi(t, x) = \psi(x)e^{i\omega t},$$

where $\omega \in R$ is the frequency and $\psi : \Omega \in R^3 \to C^4$ is the amplitude. Then the relativistic Dirac equation to the time-harmonic Dirac equation:

$$(D_{\omega,m}[\psi](x) = (i\omega\gamma_0 - \sum_{k=0}^{3} \gamma_k\partial_k + im)[\psi(x)] = 0. \tag{3}$$

**Statement of the problem.** Find a regular solutions to the time-harmonic Dirac equation (1.3)in the domain $\Omega$ using its Cauchy data on the surface $S$:

$$\psi(y) = g(y), \;\; y \in S \tag{4}$$

where $S$ is a part of the boundary of the domain, where $g(y)$ is a given continuous quaternion -valued functions on the part $S$ of boundary.

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## CAUCHY PROBLEM FOR THE QUATERNIONIC TIME-HARMONIC MAXWELL EQUATIONS

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Let $\Omega$ is a bounded simply connected domain in $R^3$ with boundary $\partial\Omega$ composed of a compact connected part $T$ of the plane $y_3 = 0$ and a smooth Lyapunov surface $S$ lying in the half-space $y_3 > 0$, with $\bar{\Omega} = \Omega \cup \partial\Omega$, $\partial\Omega = S \cup T$. Let $E$ and $H$ be the corresponding electrical and magnetic components of an electromagnetic field in $\Omega$. If an electromagnetic field $(E, H)$ is time-harmonic (or monochromatic which is a synonym)then it satisfies the following Maxwell equations [1]:

$$rot\overrightarrow{E} = i\omega\mu\overrightarrow{H}, \;\; rot\overrightarrow{H} = -i\omega\mu\overrightarrow{E}, \tag{1}$$

$$div\overrightarrow{H} = 0, div\overrightarrow{E} = 0. \tag{2}$$

Here $\omega$ is the frequency, $\epsilon$ and $\mu$ are the absolute permittivity and permeability respectively $\varepsilon = \varepsilon_0\varepsilon_r$ and $\mu = \mu_0\mu_r$, where $\varepsilon_0$ and $\mu_0$ are the corresponding parameters of a vacuum and $\varepsilon_r, \mu_r$ are the relative permittivity and permeability of a medium.

Taking into account (2) we can rewrite this system as follows

$$D\overrightarrow{E} = i\omega\mu\overrightarrow{H}, D\overrightarrow{H} = -i\omega\varepsilon\overrightarrow{E}. \tag{3}$$

This pair of equations can be diagonalized in the following way [2]. Denote

$$\overrightarrow{\varphi} := -i\omega\varepsilon\overrightarrow{E} + k\overrightarrow{H}, \tag{4}$$

$$\overrightarrow{\psi} := i\omega\varepsilon\overrightarrow{E} + k\overrightarrow{H}, \tag{5}$$

where $k := \omega\sqrt{\varepsilon\mu} = \frac{\omega}{c}\sqrt{\varepsilon_e\mu_r}$ is the wave number. Applying the operator $D$ to the functions $\overrightarrow{\varphi}$ and $\overrightarrow{\psi}$ one can see that $\overrightarrow{\varphi}$ satisfies the equation

$$(D - k)\overrightarrow{\varphi} = 0, \tag{6}$$

and $\overrightarrow{\psi}$ satisfies the equation

$$(D + k)\overrightarrow{\psi} = 0. \tag{7}$$

Solutions of (6) are (7) are called the Beltramb fiels (see, e.g., [3])

**Statement of the problem.** Find a regular solutions to the Maxwell equations (1)in the domain $\Omega$ using its Cauchy data on the surface $S$:

$$[n \times \overrightarrow{H}] = \overrightarrow{f}(y), [n \times \overrightarrow{E}] = \overrightarrow{g}(y), \ \ y \in S \qquad (8)$$

where $S$ is a part of the boundary of the domain, where $\overrightarrow{f}(y), \overrightarrow{g}(y)$ is a given continuous quaternion-valued functions on the part $S$ of boundary.

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## GENERALIZATION OF THE ESSEEN'S THEOREM


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Let

$$X_1, X_2, ...X_n, ...$$

be a sequence of nonidentically distributed independent random variables.

Consider

$$EX_j = 0, \ \ , \sigma_j^2 = EX_j^2, \quad \alpha_j = EX_j^3, \quad \beta_j = E\left|X_j\right|^3.$$

Set

$$S_n = \frac{X_1 + ... + X_n}{B_n}, \quad B_n^2 = \sum_{j=1}^{n} \sigma_j^2, \quad \Gamma_n = \sum_{j=1}^{n} \alpha_j,$$

$$F_n\left(x\right) = P\left(S_n < x\right), \ \ \Phi\left(x\right) = \frac{1}{\sqrt{2\pi}} \int\limits_{-\infty}^{x} e^{-\frac{u^2}{2}} du,$$

and introduce the following conditions:

(I) $\liminf\limits_{n\to\infty} \frac{B_n^2}{n} > 0, \limsup\limits_{n\to\infty} \frac{1}{n} \sum\limits_{j=1}^{n} \beta_j < \infty$,

(II) $\frac{1}{n} \sum\limits_{j=1}^{n} \int\limits_{|x|>n^\tau} |x|^3 \, dF_j\left(x\right) \to 0, \quad n \to \infty$ , where $\tau < \frac{1}{2}$.

We need the following condition (III) of asymptotic nonlattice of sums $S_n$, given by A.A.Borovkov [1]. For a given $\varepsilon > 0$ and $N > 0$ assuming $N > \varepsilon$ denote

$$\alpha_j\left(\varepsilon, N\right) = \max\left\{\left|f_j\left(t\right)\right|, \ \ \varepsilon \le |t| \le N\right\}.$$

We need the following condition sum $S_n$ called asymptotic nonlattice, for any fixed $\varepsilon > 0$ and $N > 0$

(III) $B_n \prod\limits_{j=1}^{n} \alpha_j(\varepsilon, N) \to 0$, as $n \to \infty$.

Our main result is the next theorem, which gives the generalization of the theorem of Esseen for a sequence of variously distributed random variables.

**Theorem.** *Let conditions (I)—(III) are satisfied, then*

$$\sup_x \left| F_n(x) - \Phi(x) - \frac{\Gamma_n}{6B_n^3}\left(1 - x^2\right)\varphi(x) \right| = o\left(\frac{1}{\sqrt{n}}\right)$$

*as* $n \to \infty$.

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## REMOVABLE SINGULARITIES OF SUBHARMONIC FUNCTIONS


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Singular sets of solutions and subsolutions Laplace equation, i.e. for harmonic and subharmonic functions are studied by several authors. Blaschke-Privalov showed the criteria of subharmonicity of functions by generalised Laplace operators [1]. I.Privalov got more deeper result with exceptional set $E$ [2]. Pokrovskii showed [3] that the well-known Blaschke-Privalov local condition selecting subharmonic functions in a given Euclidean domain $D$ from the set of real-valued upper semi-continuous functions in terms of ball means, can be replaced on some subsets of $D$ by another a priori more weak local conditions of the same type. In this work we give a criterion of the sets to be $\underline{S}$ - sets [4].

**Definition** [4]. $E$ is called $\underline{S}$ (*singular*) - set, if there exists $\nu(x) \in sh(\mathbb{R}^n) : \underline{\triangle}\nu(x)|_E = +\infty$.

$E$ is called $\overline{S}$ (*singular*) - set, if there exists $\nu(x) \in sh(\mathbb{R}^n) : \overline{\triangle}\nu(x)|_E = +\infty$.

Some properties of *singular* sets:

1. $\underline{S}$ - set is a $\overline{S}$ - set, i.e. $\underline{S}$ - sets $\subset \overline{S}$ - sets.

2. Countable union of singular $\underline{S}$ - sets is singular $\underline{S}$ - set.

3. Finite union of $\overline{S}$ - sets is singular $\overline{S}$ - set.

4. An arbitrary intersection of $\underline{S}$ - sets ( $\overline{S}$ - sets ) is again respectively $\underline{S}$ - set ( $\overline{S}$ - set).

The main results of the work are the next

**Theorem 1.** *Let a function $u(x)$, $u(x) \not\equiv -\infty$, is upper semi-continuous in the domain $D \subset \mathbb{R}^n$ and $\overline{\triangle} u \geq 0$ in $[D \setminus u_{-\infty}] \setminus E$. Then*

*a) if $E \in \underline{S}$ and $\overline{\triangle} u > -\infty$ on $E$, excluding a polar set $P \subset E \Rightarrow u(x) \in sh(D)$;*

*b) if $E \in \overline{S}$ and $\underline{\triangle} u > -\infty$ on $E$, excluding a polar set $P \subset E \Rightarrow u(x) \in sh(D)$.*

**Theorem 2.** $E \in \underline{S} \Leftrightarrow mesE = 0$.

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# APPLIED FUZZY SYSTEMS FOR PREDICTING THE PRODUCT OF A COTTON


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The quality of cotton is determined by a number of biological and technological characteristics. To determine the breeding varieties of cotton with the best biological and technological indicators used methods and algorithms for decision-making.

Below are the results of calculations for assessing the characteristics of cotton varieties.

We choose the best of the 4 breeding cotton varieties according to the following characteristics: Yield, fiber length, fiber strength, absolute seed mass, seed oil content. According to the proposed scheme, fuzzy information is expressed in the form of preference matrices $R_1, R_2, ..., R_5$ *and* $R$:

$Q_1 = R_1 \cap ... \cap R_5$ is calculated, the inverse matrix $Q_1^{-1}$ is built and $Q_1^0$is calculated:

$Q_1^{ND}$ is calculated:

$$Q_1^{ND} = \begin{bmatrix} 0,66 & 0,83 & 0,96 & 1 \end{bmatrix}.$$

Intersection is built $Q = Q_1^{ND} \cap Q_2^{ND}$:

$$Q = \begin{bmatrix} 0,66 & 0,83 & 0,96 & 0,93 \end{bmatrix}.$$

The results showed that the breeding variety 108-F is the best among the proposed breeding varieties of cotton.

The main approach to solving this problem is to develop a software application that implements the creation of knowledge and data base templates, as well as their means of filling in a dialog with the user. Setting the software for specific conditions of use in the selected problem area is reduced to filling the user with knowledge bases and system data in the learning process.

The results obtained can be used in forecasting, diagnostics, situational management, multivariate analysis, automatic classification and other tasks of processing expert information.

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# GLOBAL EXISTENCE IN A QUASILINEAR PARABOLIC-PARABOLIC CHEMOTAXIS SYSTEM


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The classical chemotaxis model may be first proposed by Keller and Segel to describe the directed movement of cells as a response to gradients of the concentration of a chemical response to gradients of the concentration of a chemical signal substance in the environment, where the chemical signal substance is consumed rather than produced by the cells themselves.

Recenty, there have been three important extensions of the Keller-Segel model: the first extension takes into account a nonlinear chemotactic sensitivity function [1], the second extension includes a nonlinear diffusion and the third extension incorporates a logistic term [2,3].

Let $\Omega$ be a bounded domain in $R^n (n = 1, 2, 3)$ with smooth boundary $\partial\Omega$. Denote $Q_T = \Omega \times (0, T)$ and $S_T = \partial\Omega \times (0, T)$, where $T > 0$ is a fixed time.

We consider a quasilinear parabolic-parabolic system with nonnegative initial data under Dirichlet boundary condition

$$u_t - \Delta u + \nabla \cdot (u\chi(x)\nabla v) = -au + bg(v)u \text{ in } Q_T, \tag{1}$$

$$v_t - \Delta v = k(v) - g(v)u \text{ in } Q_T, \quad (2)$$

$$u(x,0) = u_0(x), v(x,0) = v_0(x), x \in \Omega, \quad (3)$$

$$u = v = 0 \text{ on } S_T. \quad (4)$$

In this case, $u = u(x,t)$ denotes the density of the cells, $v = v(x,t)$ denotes the concentration of the oxygen, $g(v) = b_1 v/(1 + b_2 v)(b_1 > 0, b_2 > 0), k(v) = rv(1 - v/K)$.

Throughout this work we assume that

1) $\chi(u) \in C'[0,\infty), \chi(u) \equiv 0$ for $u \geqslant u_m$ and $\chi'(u)$ is Lipschitz continuous;

2) $u_0(x) \geqslant 0, 0 \leqslant v_0(x) \leqslant K$;

3) $\partial\Omega \in C^{2+\alpha}, u_0(x), v_0(x) \in C^{2+\alpha}(\overline{\Omega}), 0 < \alpha < 1$.

The global existence and uniqueness of classical solutions to this problem are proved by the contraction mapping principles together with $L_p$ estimates and Schauder estimates of parabolic equations.

In well-known papers, the problems are investigated under Neumann boundary conditions.

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# STRUCTURE OF ESSENTIAL SPECTRA AND DISCRETE SPECTRUM OF THE ENERGY OPERATOR OF FIVE-ELECTRON SYSTEMS IN THE HUBBARD MODEL. FOURTH QUARTET STATE


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We consider the energy operator of five-electron systems in the Hubbard model and describe the structure of the essential spectra and discrete spectrum of the system for fourth quartet state. The Hamiltonian of the chosen model has the form

$$H = A\sum_{m,\gamma} a^+_{m,\gamma} a_{m,\gamma} + B \sum_{m,\tau,\gamma} a^+_{m+\tau,\gamma} a_{m,\gamma} + U \sum_m a^+_{m,\uparrow} a_{m,\uparrow} a^+_{m,\downarrow} a_{m,\downarrow}.$$

Here $A$ is the electron energy at a lattice site, $B$ is the transfer integral between electrons neighboring sites, the summation over $\tau$ means over the nearest neighbors on the lattice, $U$

is the parameter of the on-site Coulomb interaction of two electrons, $\gamma$ is the spin index, and $a^+_{m,\gamma}$, and $a_{m,\gamma}$ are the electron creation and annihilation operators at a site $m \in Z^\nu$.

**Theorem 1.** *Let $\nu = 1$ and $U < 0$. Then the essential spectrum of the fourth quartet state operator ${}^4\widetilde{H}^q_{3/2}$ is the union of seven segments and discrete spectrum of operator ${}^4\widetilde{H}^q_{3/2}$ is consists of no more one point.*

Let $\nu = 3$, $U > 0$, $\Lambda_1 = (\Lambda^0_1, \Lambda^0_1, \Lambda^0_1)$, $\Lambda_2 = (\Lambda^0_2, \Lambda^0_2, \Lambda^0_2)$.

**Theorem 2.** *a). If $U > \frac{12B\cos\frac{\Lambda^0_1}{2}}{W}$, $\cos\frac{\Lambda^0_1}{2} > \frac{1}{2}\cos\frac{\Lambda^0_2}{2}$, $\cos\frac{\Lambda^0_1}{2} > \frac{1}{2}$, or $U > \frac{6B}{W}$, $\cos\frac{\Lambda^0_1}{2} < \frac{1}{2}$, and $\cos\frac{\Lambda^0_1}{2} > \frac{1}{2}\cos\frac{\Lambda^0_2}{2}$, or $\cos\frac{\Lambda^0_1}{2} < \frac{1}{2}\cos\frac{\Lambda^0_2}{2}$, then the essential spectrum of the fourth five-electron quartet state operator ${}^4\widetilde{H}^q_{3/2}$ is consists of the union of seven segments and discrete spectrum of operator ${}^4\widetilde{H}^q_{3/2}$ is consists of no more one point.*

*b). If $\frac{6B}{W} < U \leq \frac{12B\cos\frac{\Lambda^0_1}{2}}{W}$, $\cos\frac{\Lambda^0_1}{2} > \frac{1}{2}\cos\frac{\Lambda^0_2}{2}$, and $\cos\frac{\Lambda^0_1}{2} > \frac{1}{2}$, or $\frac{12B\cos\frac{\Lambda^0_1}{2}}{W} < U \leq \frac{6B}{W}$, $\cos\frac{\Lambda^0_1}{2} > \frac{1}{2}\cos\frac{\Lambda^0_2}{2}$, and $\cos\frac{\Lambda^0_1}{2} > \frac{1}{2}$, or $\frac{6B\cos\frac{\Lambda^0_2}{2}}{W} < U \leq \frac{6B}{W}$, $\cos\frac{\Lambda^0_1}{2} < \frac{1}{2}\cos\frac{\Lambda^0_2}{2}$, and $\cos\frac{\Lambda^0_1}{2} < \frac{1}{2}$, then the essential spectrum of operator ${}^4\widetilde{H}^q_{3/2}$ is consists of the union of four segments and discrete spectrum of operator ${}^4\widetilde{H}^q_{3/2}$ is empty: $\sigma_{disc}({}^4\widetilde{H}^q_{3/2}) = \emptyset$.*

*c). If $\frac{6B\cos\frac{\Lambda^0_2}{2}}{W} < U \leq \frac{6B}{W}$, $\cos\frac{\Lambda^0_1}{2} > \frac{1}{2}\cos\frac{\Lambda^0_2}{2}$ and $\cos\frac{\Lambda^0_1}{2} < \frac{1}{2}$, or $\frac{6B\cos\frac{\Lambda^0_2}{2}}{W} < U \leq \frac{12B\cos\frac{\Lambda^0_1}{2}}{W}$, $\cos\frac{\Lambda^0_1}{2} > \frac{1}{2}\cos\frac{\Lambda^0_2}{2}$ and $\cos\frac{\Lambda^0_1}{2} < \frac{1}{2}$, or $\frac{12B\cos\frac{\Lambda^0_1}{2}}{W} < U \leq \frac{6B\cos\frac{\Lambda^0_2}{2}}{W}$, $\cos\frac{\Lambda^0_1}{2} < \frac{1}{2}\cos\frac{\Lambda^0_2}{2}$ and $\cos\frac{\Lambda^0_1}{2} < \frac{1}{2}$, then the essential spectrum of operator ${}^4\widetilde{H}^q_{3/2}$ is consists of the union of two segments and discrete spectrum of operator ${}^4\widetilde{H}^q_{3/2}$ is empty: $\sigma_{disc}({}^4\widetilde{H}^q_{3/2}) = \emptyset$.*

*d). If $0 < U \leq \frac{6B\cos\frac{\Lambda^0_2}{2}}{W}$ and $\cos\frac{\Lambda^0_1}{2} > \frac{1}{2}\cos\frac{\Lambda^0_2}{2}$, $\cos\frac{\Lambda^0_1}{2} > \frac{1}{2}$, or $\cos\frac{\Lambda^0_1}{2} < \frac{1}{2}$, or $0 < U \leq \frac{12B\cos\frac{\Lambda^0_1}{2}}{W}$ and $\cos\frac{\Lambda^0_1}{2} < \frac{1}{2}\cos\frac{\Lambda^0_2}{2}$, $\cos\frac{\Lambda^0_1}{2} < \frac{1}{2}$, then the essential spectrum of the fourth five-electron quartet state operator ${}^4\widetilde{H}^q_{3/2}$ is single segment and discrete spectrum of operator ${}^4\widetilde{H}^q_{3/2}$ is empty: $\sigma_{disc}({}^4\widetilde{H}^q_{3/2}) = \emptyset$.*

## PROPERTIES OF $A$-LEMNISCATE


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Studying quasiconformal transformation directly related to $A-$ analytic functions. The theory of analytic solutions of Beltrami equation

$$\frac{\partial f}{\partial \bar{z}} = A(z)\frac{\partial f}{\partial z}. \quad (1)$$

The solutions of (1) equation is called $A-$ analytic functions. Denote by $\mathcal{O}_A(D)$ the class of $A-$ analytic functions with domain $D \subset \mathbf{C}$ any bounded convex domain.

Let $A(z)$ be anti-analytic function $\frac{\partial A}{\partial \bar{z}} = 0$ and for every $z \in D$ $|A(z)| \leq c < 1$.

Note that for $A-$ analytic functions, which $A(z)$ satisfying above conditions fundamental results as analog of analytic functions have been taken. For instance analogue of Cauchy's theorem and formula [1], Taylor and Laurent expansion [2], the rule of argument [3] and the theorem of Rushe [3]. Consider

$$\psi(z,\xi) = z - \xi + \overline{\int\limits_{\gamma(\xi,z)} \bar{A}(\tau)d\tau}, \qquad (2)$$

where $z, \xi \in D$ and $\gamma(\xi, z)$ is a smooth curve which connects the $\xi$ and $z$. It can be see easily that $\psi(\xi, z)$ is $A-$ analytic in $D$. The function $\psi(z, \xi)$ carries out an internal mapping. In particular, the set

$$L(\xi,r) = \left\{ z \in D : \, |\psi(z,\xi)| = \left| z - \xi + \overline{\int\limits_{\gamma(\xi,z)} \bar{A}(\tau)d\tau} \right| \right\} \qquad (3)$$

is open in $D$ and it is called $A-$ lemniscate with center $\xi$ and radius $r$, and denoted by $L(\xi, r)$. In most of above works the function (2) and $A-$ lemniscate (3) took key role.

So that investigating the function (1) and the set (3) is very important to understand geometric properties of $A-$ analytic functions.

In this paper we study some properties of $A-$ lemniscate.

**Theorem 1.** *Let $D \subset \mathbf{C}$ is a domain and $G \subset D$ is a convex subdomain. If there exist $L(a, R) \subset G$ $A-$ lemniscate, following relations are true:*
*a) for any $r, 0 < r < R$, $L(a, r) \subset G$,*
*b) for $0 < r_1 < r_2 < R$, $\partial L(a, r_1) \cap \partial L(a, r_2) = \emptyset$.*

**Theorem 2.** *Let $D \subset \mathbf{C}$ is a domain, $G \subset D$ is a convex subdomain and $A(z) \not\equiv 0$. If there exist $L(a, R) \subset G$ $A-$ lemniscate, then there exist $c(A)$ constant for any $r \in (0, R)$ following relation is hold*

$$\sup_{z_1, z_2 \in L(a,r)} |z_1 - z_2| \geq c(A).$$

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# OPTIMIZATION PROBLEM OF CONTROLLING THE HEAT PROPAGATION ON THE PARALLELEPIPED

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Let the heat propagation on the parallelepiped is described by:

$$\chi\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)+f(x,y,z)=0,\ (x,y,z)\in P \tag{1}$$

with boundary condition

$$\left.\left(\chi\frac{\partial u}{\partial \vec{n}}+\alpha u\right)\right|_{\partial P}=0; \tag{2}$$

$$m(x,y,z)\le u(x,y,z)+T_0\le M(x,y,z), \tag{3}$$

here $P=\{(x,y,z): a_k\le x_k\le b_k,\ k=1,2,3\}$, $\vec{n}-$ outer normal to sides of the parallelepiped $P$, $\chi-$ coefficient of thermal diffusivity of the medium, $\alpha-$ coefficient of heat transfer through the boundary $\partial P$, $u(x,y,z)=T(x,y,z)-T_0$ the difference between the temperature inside $P$ and the temperature $T_0$ of the environment, $m(x,y,z), M(x,y,z)$ set in the $P$ region the minimum and maximum temperature profiles, which are considered to be continuous functions. The density of heat sources $f(x,y,z)$ is assumed to belong to the space $L_2(P)$ of square integrable functions.

The problem can be reformulated (1)-(3) as the minimum problem for the functional $J(f)=\int\limits_P f(x,y,z)dxdydz$ under the following conditions on the density of sources:

$$f(\cdot,\cdot)\in L_2(P);\ m(x,y,z)-T_0\le-\frac{1}{\chi}(Gf)(x,y,z)\le M(x,y,z)-T_0, \tag{4}$$

where $G-$ Green's function. We construct a finite-dimensional approximation of the problem (1)-(4) in the form of a linear programming problem.

Substituting the expression for $f(x,y,z)$ into (1) and multiplying scalarly into $L_2(P)$ inequalities in (4) by $e_j(x,y,z)$ we get linear programming problem [1]

$$\begin{aligned} &J_n\{f\}=\sum_{j=1}^{n}(mesP_j)f_j\to\min,\\ &a_i\le\sum_{j=1}^{n}a_{ij}f_j\le b_i,\ f_i\ge 0\ \ (i=1,2,...,n).\end{aligned} \tag{5}$$

In order to solve the problem (5), the linear programming-simplex method was applied. The corresponding algorithm and program were compiled, with a support numerical results. The program is written in the algorithmic language C#.

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## MULTIPLE PURSUIT PROBLEMS WITH DIFFERENT CONSTRAINTS IN THE COORDINATES OF CONTROL PARAMETERS

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In this work, we study a differential game of many pursuers and one evader, the motions of all players are simple. A geometric constraint imposed on one part of the control parameters of the players, the rest is an integral constraint. Sufficient conditions for multiple prosecution were given.

We study a pursuit differential game of $m$ pursuers $P_i$, $i = 1, ..., m$ and one evader $E$ which is described by the following differential equations

$$\begin{aligned} &\dot{x}_i = u_i, \quad x_i(0) = x_{0i}, \; i = 1, ..., m, \\ &\dot{y} = v, \quad y(0) = y_0, \end{aligned} \tag{1}$$

where $x_i$, $y$, $u_i$, $v \in \mathbb{R}^n$, $x_{0i} \neq y_0$, $P_i-$ control parameter of the pursuer $u_i = (u_{1i}, \dots, u_{ni})$, $i = 1,\ 2,\ ...,\ m$, and $- v = (v_1, \dots, v_n)$ is that of the evader $E$.

Assume controls of the pursuers and evader is defined as the measurable function $u_i(t) = (u_{1i}(t), \dots, u_{ni}(t))$, $v(t) = (v_1(t), \dots, v_n(t))$, $t \geq 0$ subjected to contraints:

$$\sum_{j=1}^{k} |u_{ji}(t)|^2 \leq \rho_{1i}^2, \; t \geq 0, \; \int_0^\infty \sum_{j=k+1}^{n} |u_{ji}(t)|^2 dt \leq \rho_{2i}^2, \quad i = 1, \dots, m, \tag{2}$$

$$\sum_{j=1}^{k} |v_j(t)|^2 dt \leq \sigma_1^2, \; t \geq 0, \int_0^\infty \sum_{j=k+1}^{n} |v_j(t)|^2 dt \leq \sigma_2^2, \tag{3}$$

where $\rho_{1i}$, $\rho_{2i}$, $\sigma_1$, $\sigma_2-$ positive numbers.

**Assume.** There are sets $Q_j \subset Q$, $j = 1, ..., d$ satisfying condition $Q_i \cap Q_j = \emptyset$, $i \neq j$ for set $Q = \{i : \; \sigma_1 < \rho_{1i}, \; i \in \{1, ..., m\}\}$ and $d \leq m$, let it satisfy

$$S = \left\{ (\sigma_1, \sigma_2) : \sigma_2^2 < \sum_{i \in Q_j} \rho_{2i}^2, \; j = 1, ..., d \right\} \neq \emptyset$$

.

**Theorem 1.** *If assumption fullfiled, optional* $(y_0,\ x_{01},\ x_{02},\ ...,\ x_{0m})$ *the starting position in the game (1) - (3) happen keeps d.*

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# ON THE CONTINUATION OF THE SUM OF THE HARTOGS SERIES

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In this paper we consider the question of continuation of the sums of the Hartogs series that admit holomorphic continuation along a fixed direction with "thin" singularities, assuming only the holomorphicity of the coefficients of the series and investigate the region of convergence of such series.

The main result of the work is the following theorem.

**Theorem 1.** *Let the series*

$$f(z,w) = \sum_{k=0}^{\infty} c_k(z)\, w^k, \tag{1}$$

*satisfies the following conditions:*

*1) $c_k(z) \in O(D)$, $k = 0, 1, 2, \ldots$; where $D \subset \mathbb{C}_z^n$ is a domain,*

*2) for each fixed $z \in D$ the sum of the series (1), as a function of a variable $w$, extends holomorphically to the whole plane $\mathbb{C}_w$, except some polar (discrete) set $P_z \subset \mathbb{C}\backslash\{w = 0\}$.*

*Then there exists nowhere dense closed set $S \subset D$ and pluripolar (analytic) set $P$ in $(D\backslash S) \times \mathbb{C}$, such that the series (1) defines a holomorphic function $f(z,w)$ in $[(D\backslash S) \times \mathbb{C}] \setminus P$.*

Note that the presence $S$ in this theorem is necessary.

From Theorem 1 and from Sadullayev—Chirka's theorem [1] it follows that a more general theorem holds.

**Theorem 2.** *Let the series (1) be such that:*

1) $c_k(z) \in O(D)$, $k = 0, 1, 2, \ldots$;

2) *for each fixed point $z \in D$ the radius of convergence of the series (1) is $R(z) > 0$,*

3) *for each fixed $z \in E$ of some open set $E \subset D$ the sum of the series (1), as a function of $w$, extends holomorphically to the whole plane $\mathbb{C}_w$, with the exception of some polar (discrete) set $P_z$.*

*Then there exists nowhere dense closed set $S \subset D$ and pluripolar (analytic) set $P$ in $(D\backslash S) \times \mathbb{C}$ such that the sum of the series (1) is holomorphic function $f(z,w)$ in $[(D \setminus S) \times \mathbb{C}] \setminus P$.*

**Remark.** If in Theorem 2 we do not require the condition $R(z) > 0$ for any fixed $z \in D$, then the statement of the theorem is not true, i.e. condition 2 of Theorem 2 is necessary.

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# MEAN ERGODIC THEOREM WITH CONTINUOUS TIME IN FUNCTION SYMMETRIC SPACES FOR INFINITE MEASURE

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Let $S = ((0, +\infty), \nu)$ be a measure space with a complete $\sigma$-finite Lebesgue measure $\nu$, and let $L_0 = L_0(S)$ be the algebra of a.e. finite real-valued measurable functions on $S$ (equal a.e. functions are identified). Let $L_0(\nu)$ be the subalgebra in $L_0$ consisting of functions $f \in L_0$ such that $\nu(\{|f| > \lambda(f)\}) < \infty$ for some $\lambda(f) > 0$. If $f \in L_0(\nu)$, then a *non-increasing rearrangement* of the function $f$ is defined as

$$f^*(t) = \inf\{\lambda > 0 : \ \nu(\{|f| > \lambda\}) \leqslant t\}, \quad t > 0.$$

A non-zero linear subspace $E$ in $L_0(\nu)$ with the Banach norm $\|\cdot\|_E$ is called *fully symmetric* space on $S$ if the conditions $f \in E, \ g \in L_0(\nu), \ \int\limits_0^s g^*(t)dt \leqslant \int\limits_0^s f^*(t)dt$ for all $s > 0$, imply that $g \in E$ and $\|g\|_E \leqslant \|f\|_E$. Examples of fully symmetric spaces are classic Banach spaces $L_p = L_p(S)$, $1 \leqslant p \leqslant \infty$, equipped with the norm $\|\cdot\|_p$.

A linear operator $T : L_1 + L_\infty \to L_1 + L_\infty$ is called a *Dunford-Schwartz operator* (writing $T \in DS$) if $\|T(f)\|_1 \leqslant \|f\|_1$ for all $f \in L_1$ and $\|T(f)\|_\infty \leqslant \|f\|_\infty$ for all $f \in L_\infty$.

It is known that every fully symmetric space $E$ is an exact interpolation space for the Banach couple $(L_1, L_\infty)$, in particular, $T(E) \subseteq E$ and $\|T\|_{E\to E} \leqslant 1$ for any $T \in DS$.

Let $\{T_t\}_{t\geqslant 0} \subset DS$ be a strongly continuous semigroup in $L_1$, that is, $\|T_t(f) - T_{t_0}(f)\|_1 \to 0$ whenever $t \to t_0$ for all $f \in L_1$. Then, given $f \in L_1$ and $t > 0$, there exists $A_t(f) = \frac{1}{t}\int_0^t T_s(f)ds \in L_1$, in addition, the linear operator $A_t : L_1 \to L_1$ is contraction in $(L_1, \|\cdot\|_1)$ and $\|A_t(f)\|_\infty \leqslant \|f\|_\infty$ for all $f \in L_1 \cap L_\infty$. Consequently, the operator $A_t$ has a unique extension to a Dunford-Schwartz operator [1, Proposition 1.1], which we also denoted by $A_t$.

The well-known mean ergodic theorem for strongly continuous in $L_1$ semigroup $\{T_t\}_{t\geqslant 0} \subset DS$ asserts (see, for example, [3, Chapter VIII, §7]) that the averages $A_t(f)$ converge strongly in $L_p$, $1 < p < \infty$, that is, given $f \in L_p$, there exists $\widehat{f} \in L_p$ such that $\|A_t(f) - \widehat{f}\|_p \to 0$ as $t \to +\infty$. For the spaces $L_1$ and $L_\infty$, the mean ergodic theorem is false, in general.

The following theorem is the criterion for the validity of the mean ergodic theorem for strongly continuous in $L_1$ semigroups, acting in a fully symmetric spaces (cf [2], [4]).

**Theorem.** *Let $(E, \|\cdot\|_E)$ be a fully symmetric space on $S = ((0, \infty), \nu)$. Then the following conditions are equivalent:*

*$(i)$. For any strongly continuous in $L_1$ semigroup $\{T_t\}_{t\geqslant 0} \subset DS$ and for any $f \in E$ there exists $\widehat{f} \in E$, such that $\|A_t(f) - \widehat{f}\|_E \to 0$ as $t \to +\infty$;*

*$(ii)$. $(E, \|\cdot\|_E)$ is separable space and $E \nsubseteq L_1$ as sets.*

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## THE INVERSION THEOREM FOR TRUNCATED BALL FRACTIONAL DERIVATIVES

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Let $0 < c < \infty$ be any fixed point and $\alpha > 0$. We refer to

$$\left(B_c^{\alpha}\varphi\right)(x) := \begin{cases} \gamma_{n,\alpha} \int\limits_{(c,|x|)} \frac{\left(|x|^2-|y|^2\right)^{\alpha}}{|x-y|^n}\varphi(y)\,dy, & |x| > c, \\ \gamma_{n,\alpha} \int\limits_{(|x|,(c)} \frac{\left(|y|^2-|x|^2\right)^{\alpha}}{|x-y|^n}\varphi(y)\,dy, & |x| < c, \end{cases}$$

as the Chen-type modification of the ball fractional integrals of order $\alpha$.

Chen-type ball fractional derivatives of the order $\alpha$ and ball layer $U(a,b)$, $0 \le a < c < b \le \infty$, we define as

$$D_c^{\alpha} f = \frac{1}{\chi(\alpha,l)} \int\limits_0^{\infty} \frac{\left(\Delta_{\tau}^{l} A_x^{+} f_{c+}\right)(0) + \left(\Delta_{\tau}^{l} A_x^{-} f_{c-}\right)(0)}{\tau^{1+\alpha}}\, d\tau,$$

where $l > \alpha > 0$ and $\chi(\alpha,l) = \int_0^{\infty}\left(1-e^{-t}\right)^l t^{-1-\alpha}dt$ -is a normalizing factor,

$$\left(\Delta_{\tau}^{l} A_x^{+} f_{c+}\right)(0) = f_{c+}(x) +$$

$$+\sum_{j=1}^{l}(-1)^j \begin{pmatrix} l \\ j \end{pmatrix} \left(1-\frac{j\tau}{r^2}\right)_+^{\frac{n}{2}-1} \Pi[f_{c+}\left(\sqrt{r^2-j\tau},\cdot\right)]\left(\sqrt{1-\frac{j\tau}{r^2}},x'\right),$$

$$\left(\Delta_{\tau}^{l} A_x^{-} f_{c-}\right)(0) = f_{c-}(x) +$$

$$+\sum_{j=1}^{l}(-1)^j \begin{pmatrix} l \\ j \end{pmatrix} \Pi[f_{c-}\left(\sqrt{r^2+j\tau},\cdot\right)]\left(\left(1+\frac{j\tau}{r^2}\right)^{-\frac{1}{2}},x'\right).$$

In this paper, we prove theorems for the inversion of Chen-type ball fractional integral $\left(B_c^{\alpha}\varphi\right)(x)$ from the function $L_p(U(a,b))$, truncated ball fractional derivative $\left(D_c^{\alpha} f\right)(x) := \left(B_c^{\alpha}\right)^{-1} f(x)$.

**Theorem 1.** *Let* $f = B_c^{\alpha}\varphi$, *where* $\alpha > 0,\ \varphi \in L_p\left(U\left(a,b\right)\right),\ 1 \leq p < \infty,\ 0 < a < c < b < \infty$. *Then*

$$D_c^{\alpha} f = \lim_{\varepsilon \to 0} D_{c,\varepsilon}^{\alpha} B_c^{\alpha}\varphi = \varphi,$$

*where the limit being interpreted in the* $L_p$*-norm.*

**Theorem 2.** *Let* $f = B_c^{\alpha}\varphi$, *where* $\alpha > 0,\ \varphi \in L_p\left(U\left(a,b\right)\right),\ 1 < p < \infty,\ 0 < a < c < b < \infty$. *Then*

$$D_c^{\alpha} f = \lim_{\varepsilon \to 0} D_{c,\tilde{\varepsilon}}^{\alpha} B_c^{\alpha}\varphi = \varphi,$$

*where the limit being interpreted in the* $L_p$*-norm,* $\tilde{\varepsilon} = \varepsilon \left| |x|^2 - c^2 \right|$.

## QUANTUM NETWORKS WITH REFLECTIONLESS BRANCHING POINTS

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The problem of transparent boundary conditions (TBCs) for wave equations has attracted much attention in different practically important contexts (see, e.g., papers [1-12] for review). Such boundary conditions permit of no reflection of particles and waves in their transmission from one domain to another one. Strict mathematical treatment of TBC for different wave equations, including quantum mechanical Schrödinger equation can be found in the Refs. [13-25]. According to these papers the boundary conditions can be derived by factorization of the differential operator, corresponding to a wave equation, which in general lead to complicated equations for the boundary conditions. Explicit form of such boundary conditions are much complicated than those of Dirichlet, Neumann and Robin conditions.

For transparent boundary conditions, the wave equation cannot be solved analytically and always requires using numerical methods. Depending on the type of the wave equation, they require different discretization schemes, which vary also for the types of the process.

In this work we study the problem of transparent quantum networks modeled by quantum graphs [26-28], determining them as branched quantum wires providing reflectionless transmission of waves at the branching points. Within our approach we define TBCs by the fact that they should lead to an initial boundary value problem with a unique solution in some suitable $L^2$ space that coincides with the corresponding solution on the unbounded/infinite graph. Using such approach, we showed that Kirchhoff-type boundary conditions (weight continuity and current conservation) can become equivalent to transparent vertex boundary conditions under constraints in the form of a simple sum rule that is fulfilled for the weight coefficients.

We have shown numerically for the star graph a reflectionless transmission of the Gaussian wave packet through the vertices, if these constraints are fulfilled. The approach can be extended straight forward for arbitrary graph topologies, which contain any subgraph connected to two or more outgoing, semi-infinite bonds.

Finally, we note that extension of the above approach of other linear wave equations, such as heat, diffusion, Dirac and Klein-Gordon equations should be rather straightforward. Moreover, the method can be modified and applied for the nonlinear partial differential equations widely used in physics. First step on this way was done, e.g. in [29].

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## ON DIMENSION OF THE SPACE OF MONETARY RISK MEASURES

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The financial market faces risks arising from many types of uncertain losses, including market risk, credit risk, liquidity risk, operational risk, etc. (for details, see [1]). Let $X$ be a compact Hausdorff space, $C(X)$ be the Banach algebra of continuous maps. The algebra $C(X)$ plays a role of the set of all potential risks. A *risk measure* is a numerical value $\mu(\varphi)$ to quantify the risk of a potential loss $\varphi \in C(X)$. A map $\mu$ from $C(X)$ to $\mathbb{R}$ is called a *monetary risk measure*, if it satisfies two conditions: (1) *monotonicity*: for all $\varphi, \psi \in C(X)$ satisfying $\varphi \leq \psi$, it holds that $\mu(\varphi) \leq \mu(\psi)$; (2) *translation invariance*: for all $\varphi \in C(X)$ and any real number $\alpha$, it holds that $\mu(\varphi + \alpha_X) = \mu(\varphi) + \alpha$. A monetary risk measure $\mu\colon C(X) \to \mathbb{R}$ is called *normed* if (3) $\mu(1_X) = 1$. In works [2] – [17] the set of all normed monetary risk measures is denoted by $O(X)$, and it is established a lot of its topological properties. In this mark we give an example showing that the space $O(X)$ of monetary risk measure does not have infinite algebraic dimension if $X$ consist more than one point.

**Example 1.** Let $X = \{0, 1\}$ be a discrete two-point space. Then $C(X) = \mathbb{R}^2$. Each functional $\mu\colon C(X) \to \mathbb{R}$ defined by the equality

$$\mu(\varphi) = \alpha_1 \varphi(0) + \alpha_2 \varphi(1) + \alpha_3 \max\{\varphi(0) + \lambda_1, \varphi(1) + \lambda_2\} + \\ + \alpha_4 \min\{\varphi(0) + \lambda_3, \varphi(1) + \lambda_4\} + \alpha(\varphi) f(\varphi(1) - \varphi(0)) \quad (1)$$

is a normed monetary risk measure. Here $\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 = 1$ with $\alpha_i \geq 0$, $i = 1, 2, 3, 4$, $\lambda_1, \lambda_2 \in [-\infty, 0]$ with $\max\{\lambda_1, \lambda_2\} = 0$, $\lambda_3, \lambda_4 \in [0, +\infty]$ with $\min\{\lambda_3, \lambda_4\} = 0$,

$$\alpha(\varphi) = \begin{cases} \min\{\alpha_1, \alpha_2\}, & \text{if } \alpha_3 = \alpha_4 = 0, \\ \min\{\alpha_1 + \alpha_3 + \alpha_4, \alpha_2\}, & \text{if } \varphi(0) + \lambda_1 \geq \varphi(1) + \lambda_2 \text{ and } \varphi(0) + \lambda_3 \leq \varphi(1) + \lambda_4, \\ \min\{\alpha_1 + \alpha_3, \alpha_2 + \alpha_4\}, & \text{if } \varphi(0) + \lambda_1 \geq \varphi(1) + \lambda_2 \text{ and } \varphi(0) + \lambda_3 > \varphi(1) + \lambda_4, \\ \min\{\alpha_1 + \alpha_4, \alpha_2 + \alpha_3\}, & \text{if } \varphi(0) + \lambda_1 < \varphi(1) + \lambda_2 \text{ and } \varphi(0) + \lambda_3 \geq \varphi(1) + \lambda_4, \\ \min\{\alpha_1, \alpha_2 + \alpha_3 + \alpha_4\}, & \text{if } \varphi(0) + \lambda_1 < \varphi(1) + \lambda_2 \text{ and } \varphi(0) + \lambda_3 > \varphi(1) + \lambda_4, \end{cases} \quad (2)$$

and, finally, $f\colon \mathbb{R} \to \mathbb{R}$ is a continuous non-decreasing function such that $(1^*)$ $f(0) = 0$, $(2^*)$ $t \le f(t) \le 0$ and concave at $t \le 0$, $(3^*)$ $0 \le f(t) \le t$ and convex at $t \ge 0$.

**Remark** As among the functions $f$ considered in (2) and satisfying conditions $(1^*)$ – $(3^*)$, there is an uncountable linearly independent system, it follows that the space $O(X)$ of normed monetary risk measure does not embed in any space with finite (even countable) algebraic dimension as soon as the compact $X$ contains more than one point.

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## VECTOR-VALUED REARRANGEMENTS OF MEASURABLE FUNCTIONS

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In this note are considered rearrangements of the elements of the algebra $C_\infty(Q(B))$ for Maharam measure that takes values in the vector lattices $L^0(\Omega)$ of all equivalence classes of real measurable functions on $(\Omega, \mathcal{A}, \mu)$, where $(\Omega, \mathcal{A}, \mu)$ is measure space with a complete $\sigma$-finite measure $\mu$.

Let $B$ be an arbitrary complete Boolean algebra, let $Q(B)$ be a Stone compact corresponding to $B$, and let $L^0(B) = C_\infty(Q(B))$ be an algebra of all continuous functions $x : Q(B) \to [-\infty, +\infty]$, taking the values $\pm\infty$ only on nowhere dense sets. The strongly positive completely additive measure $m : B \to L^0(\Omega)$ is called a Maharam measure if for any $e \in B$, $0 \le f \le m(e)$, $f \in L^0(\Omega)$ there exists $q \in B$, $q \le e$, such that $m(q) = f$. By [1, Proposition 3.2], the complete Boolean algebra $\mathbf{B}(\Omega)$ of all idempotents in $L^0(\Omega)$ is identified with a regular Boolean subalgebra $A$ in $B$. In this case the algebra $L^0(\Omega)$ is identified with a subalgebra $L^0(A) = C_\infty(Q(A))$ in $L^0(B)$.

An element $x \in L^0(B)$ is called *A-bounded,* if there exists the element $0 \le a \in L^0(A)$ such that $|x| \le a$. The set $L^\infty(B, A)$ of all $A$-bounded elements from $L^0(B)$ is a subalgebra in $L^0(B)$. For any $x \in L^\infty(B, A)$ we set $\|x\|_A = \inf\{0 \le a \in L^0(A) : |x| \le a\}$. If $\psi$ is an isomorphism from $L^0(A)$ onto $L^0(\Omega)$, then the mapping $\|x\|_{\infty,B} = \psi(\|x\|_A)$ defines the $L^0$-valued norm on $L^\infty(B, A)$.

**Definition.** A mapping $n_x : (0, \infty) \to L^0(\Omega)$ is called non-increasing $m$-rearrangement of the element $x \in L^0(B)$, if

$$n_x(t) = \inf\{\|xp\|_{\infty,B} : p \in A, m(\mathbf{1}_B - p) \le t\mathbf{1}\},\ t > 0,\ \text{where } \mathbf{1}_B \text{ unit in } B.$$

Consider the product $((0, +\infty), \Sigma, \nu) \otimes (\Omega, \mathcal{A}, \mu)$ of measure spaces, where $\Sigma$ is the $\sigma$-algebra of Lebesgue measurable sets from $(0, +\infty)$ and $\nu$ is a Lebesgue measure on $\Sigma$. Define the non-negative measurable function $m_x(t, \omega)$ on $((0, +\infty), \Sigma, \nu) \otimes (\Omega, \mathcal{A}, \mu)$ as $m_x(t, \omega) = \inf\{\tau > 0 : m\{|x| > \tau\}(\omega) \le t\}$, $\omega \in \Omega$, $t > 0$.

**Proposition 1.** *If $x \in L^0(B)$ then for every $t > 0$ the equality $n_x(t)(\omega) = m_x(t, \omega)$ follows for almost everywhere $\omega \in \Omega$.*

The following formula gives a method for calculating non-increasing $m$-rearrangements.

**Proposition 2.** *Let $t > 0$ and $\mathcal{R}_t = \{z \in L^0(B) : m(s(|z|)) \le t\mathbf{1}_{\mathbf{B}(\Omega)}\}$, where $s(|z|)$ is support of an element $|z|$. Then*

$$n_x(t) = \inf\{\|x - z\|_{\infty,B} : z \in \mathcal{R}_t,\ x - z \in L^\infty(B, A)\}.$$

*for any $x \in L^0(B)$.*

Let $L^1(B, m)$ be the linear space of all integrable functions from $L^0(B)$ with respect to the Maharam measure $m$. The mapping $\|x\|_1 = \int\limits_B |x|\, dm,\ x \in L^1(B, m)$, is the $L^0(\Omega)$-valued norm on $L^1(B, m)$ such that $(L^1(B, m), \|\cdot\|_1)$ is Banach-Kantorovich space (see [2], п.6.1.10).

**Theorem.** *If $x \in L^1(B, m)$ then $\|x\|_1(\omega) = \int\limits_0^\infty m_x(t, \omega)dt$ for a.e. $\omega \in \Omega$.*

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## ON MIXED NONLOCAL PROBLEM WITH INTEGRAL CONDITION FOR THE LOADED PSEUDOPARABOLIC EQUATION


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In this paper we consider loaded partial differential equation of the third order

$$Mu \equiv Lu + \int\limits_0^l k(x,t)u(x,t)dx = f(x,t), \tag{1}$$

where $Lu \equiv u_{xxt} + a(x,t)u_{xx} + b(x,t)u_{xt} + c(x,t)u_x + d(x,t)u_t + e(x,t)u-$ pseudoparabolic operator, $k(x,t)$ and $f(x,t)$ are given functions.

We consider following problem A:

*In the domain $D = \{(x,t) : 0 < x < l,\ 0 < t < T\}$ find solution $u(x,t)$ of equation (1), which satisfies initial condition*

$$u(x, 0) = \varphi_0(x), \qquad 0 \le x \le l, \tag{2}$$

*the integral condition*

$$u(0,t) = \beta(t)\int\limits_0^l u(x,t)dx + \int\limits_0^t \rho(t,\tau)u(l,\tau)d\tau + \psi_1(t), \qquad 0 \le t \le T, \tag{3}$$

*and the Neumann boundary condition*

$$u_x(0,t) = \psi_2(t), \qquad 0 \le t \le T, \tag{4}$$

where $\varphi_0(x)$, $\psi_1(t)$, $\psi_2(t)$, $\beta(t)$, $\rho(t,\tau)$ are given functions such that

$$\varphi_0(0) = \beta(0)\int_0^l \varphi_0(x)dx + \psi_1(0), \qquad \varphi_0'(0) = \psi_2(0).$$

Problem A we consider in space $C^{2,1}(D)\bigcap C^{1,0}(\overline{D})$ and in this case we require execution of following conditions:

**Condition 1.** Coefficients of the equation (1) for all $(x,t) \in D$ are satisfies following conditions

$$a \in C^{1,0}(\overline{D}) \cap C^{2,0}(D); \quad b \in C^1(\overline{D}) \cap C^{1,1}(D);$$

$$c \in C(\overline{D}) \cap C^{1,0}(D); \quad d \in C(\overline{D}) \cap C^{0,1}(D); \quad e \in C(D).$$

Furthermore, $d < 0$ for any $(x,t) \in D$.

**Condition 2.** Given functions $\varphi_0(x)$, $\psi_1(t)$, $\psi_2(t)$, $\beta(t)$, $f(x,t)$, $k(x,t)$ and $\rho(t,\tau)$ satisfy conditions

$$\varphi_0(x) \in C^2[0,l]; \ \psi_1(t), \ \psi_2(t) \in C^1[0,T]; \ \beta(t), \ \rho(t,\tau) \in C[0,T],$$

$$f(x,t), \ k(x,t) \in C(\overline{D}); \quad \beta(t) < 0, \quad \forall t \in [0,T].$$

The following is valid

**Theorem.** *Let Condition 1 and Condition 2 are fulfilled. Then problem A has unique classical in* $\overline{D}$ *solution.*

# BIOLOGY AND MEDICINE

## Plenary Lectures

### SYNTHETIC RNAi DUPLEX OF COTTON (GOSSYPIUM HIRSUTUM L) PHYTOCHROME B GENE ENHANCES MULTIPLE AGRONOMIC TRAITS

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**Abstract.** Different light-sensing systems involve responding the light signal. The majority of important plant responses are regulated by the phytochrome photoreceptor system. Previously, RNAi technology was used to characterize *PHYA1* gene function in cotton, and several high-quality novel cotton cultivars were developed. Here, we generated synthetic oligonucleotide-based RNAi duplexes in order to specifically silence of phytochrome B gene (*PHYB*) in cotton. The overall objectives of this research were to: 1) create synthetic oligonucleotide-based duplex for cotton *PHYB* gene(s); 2) develop and characterize new biotechnological cotton cultivars based on *SynB* RNAi lines; 3) obtain high generations of *SynB* RNAi lines by self-pollination and crossing with Uzbek commercial cultivars; 4) study morphological traits within greenhouse and field condition over in multiple years/seasons; and 5) statistically analyze greenhouse and field agronomic performance data. As results, a number of somatically regenerated *SynB* RNAi lines of Cocker-312 were obtained with increased fiber strength, low micronaire (finer fiber) and decreased flowering time. Novel *SynB* RNAi lines were further crossed with several elite Uzbek commercial Upland cultivars. Result from independent plants of three generations of *SynB* RNAi and their hybrids ($BC_3F_3$) showed significant changes in fiber micronaire and an improvement of other major traits, such as vigorous vegetative growth and early-flowering. Our results should be useful for the development of early-maturing and superior fiber quality Upland cultivars.

**Introduction**

Cotton (*Gossypium spp.*) is a natural fiber source for the textile industry, and one of the most important crops in many countries. Its product is around 25 million tons annually. The average yield of World cotton was improved to 9% to 753 kg/ha in 2016/17 years (www.icac.org). However, to increase fiber quality, yield and early maturity is a difficult task without modern approaches. Moreover, genetically enrichment of crop varieties using modern transgenomics, tissue culture and genetic engineering methods is essential task to breeders. Improvement of productivity, fiber quality and tolerance to soil-climatic conditions of the world cotton varieties are closely linked to the high level of technology in breeding. One of the genome editing technologies RNA interference technology has been developed to increase agricultural crops significantly. The numbers of genes, including *PHYA*, *MYB* and *CryA/B* genes have been studied and being used in agricultural purposes by this technology. Special attention is paid to the development of intensive and innovative ideas in agricultural production using of new biotechnology methods, and the creation of new varieties of crops, their productivity and disease tolerance in our country. As results many genes associated with fiber quality, early flowering and productivity were cloned and detail characterized. Early maturity and high quality new cotton varieties were developed. RNA interference of flowering regulator genes using synthetic oligonucleotides will be valuable approach in the future to enrich cotton varieties and supply with competitive product of the World market.

*PHYB* gene is a photoreceptor that detects the shadow of the plant and is involved in plant protection, and inhibits the flowering process in daylight. On the mutant plants of the sorghum (*Sorghum bicolor* L.) and pisum (*Pisum sativum*) that lost the *PHYB* gene function, scientists have conducted a series of research. When phyb mutations occurred, those plants showed early flowering than wild type and decreased photoperiodic sensitivity (Lin, 2000).

In Uzbekistan many researches have been implemented in this direction and all phytochrome genes were cloned and characterized from diploid and allotetraploid cotton species (Abdurakhmonov et al., 2010). For the first time, new biotechnological cotton varieties have been developed based on *PHYA1* gene RNA interference with positive characteristics, such as early flowering and maturity, high yield and fiber quality (Abdurakhmonov et al. 2014). Moreover, synthetic oligonucleotide duplexes were synthesized for *A. thaliana* phytochrome genes by Abdukarimov et al., 2011. In addition, RNA interference of cotton *PHYB* gene in order to study gene functions have been accomplished.

**Material and methods**

**Plant materials.** a) To transfer SynB RNAi genetic construction model plant Arabidopsis thaliana was used as a plant material, and RNAi effect was observed in the number of generattions; b) The somatically regenerable cotton variety Cocker-312 (G. hirsutum L.) was used to transfer RNAi construction (obtained from cotton germplasm collection of Center of Genomics and bioinformatics, Tashkent, Uzbekistan); c) Several commercially important Uzbek cotton cultivars were then used to verify the transferability of the observed RNAi effects. Including, G. hirsutum cv. AN-Boyovut-2, Shodlik-9 and Andijan-35 (obtained from cotton germplasm collection of Center of Genomics and bioinformatics, Tashkent, Uzbekistan).

**DNA isolation**. Genomic DNA was isolated from A. thaliana and the number of G. hirsutum species by the modified method (Abdurakhmonov et al., 2014).

**Polymer chain reactions.** Polymer chain reactions were implemented in 50 μl of final volumes (2.5 μl 10 x PCR buffer, 2 μl $MgCl_2$, 1μl 10% PVP, 0.5 μl 10 mM of a dNTP mix, 2.5 μl 10 ng/ml reverse and forward primer, and 1 μl 50 ng/ml templates DNA. 0.5 Unit Taq polymerase (Sigma, USA) were added to the reaction at the annealing temperature of first cycle. PCR amplification were carried out by following steps: a first denaturation at 94°C for 3 min followed by 35 cycles of 94°C for 50 sec, 50° C for 1.30 min (annealing) and 72°C for 2 min (extension). A final 7 min extension at 72°C was then performed. Later, procedure of PCR cycles was modified purposely according to amplicon sizes.

Unlike other gene silencing technologies we decided to use small interfering RNA technique based on synthetic oligonucleotide duplexes for interested gene. For this purpose, a binary vector system was designed to use in *Agrobacterium*-mediated plant transformation. This vector system contains the expression cartridge of the primary cloning vector, *pART7*, cauliflower mosaic virus CaMV35S promoter, a multiple cloning site and the OCS termination region of the gene. Several primer pairs were then designed and synthesized to amplify genetic construct portion from bacterial and plant cells as well as to check correct insertion of genetic construct.

From the nucleotide sequence of *G. hirsutum PHYB* gene, the strictly specific short regions in the size of 19-21 from the *PHYB* were selected based on these specific sequences, a synthetic duplex was constructed and then the duplex was chemically synthesized from IDT (Integrated DNA Technologies, USA). The sequence of the synthetic duplex of the *PHYB* gene is 65 bp long in which 21 bp is sense and 21 bp is the antisense, located in the reverse order. There is an intron in the length of 9 bp (TTGAACTCT) between the two sequences - the sense and antisense. At the end of the duplex is a terminating sequence - T repeat in length of 6 bp (TTTTTT). The two ends of the fragment are flanked by *XhoI* restriction sites at a length of 6 bp (CTCGAG) and *XbaI* in a length of 6 bp (TCTAGA).

In this reaction single-stranded primers were converted to double stranded structure using Potassium acetate buffer. Duplexed product was incubated at 95$^0$C for 5 minutes and then kept in the freezer until next procedure.

The duplex was inserted into the binary vector *pART27* to check the expression of the synthetic duplex. This vector contains two restriction sites (*XhoI* and *XbaI)* between the CamV35S promoter and the OCS terminator, as well as the kanamycin resistance gene for plant breeding and the spectinomycin resistance gene for bacterial selection. The fusion of the prepared *pART27* plasmid with the synthetic duplex was ligated in the following way: 1 µg (200 ng/µl) of the purified plasmid *pART27*, 50 µg synthetic duplex, 10 mM ATP, ligated buffer, 0.1 units. T4 DNA ligase (Invitrogene, USA) was mixed and incubated at 22° C for 16 hours (Figure 1).

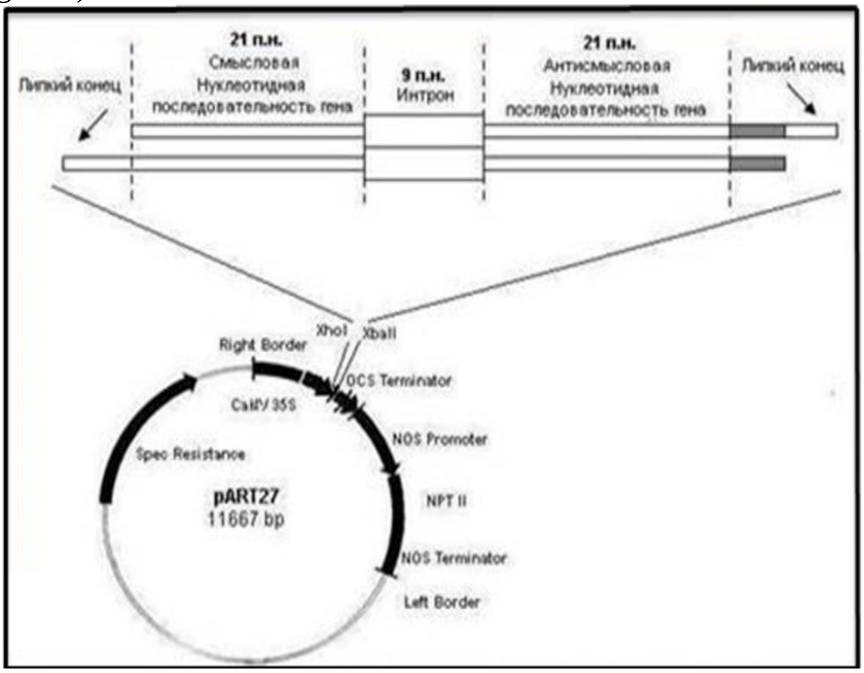


*Figure 1. Schematic structure of SynB RNAi genetic construction.*

*pART27* binary vector along with genetic construct was transformed into *Esherihia coli* cells (DH5@ strain). *E. coli* cells were then placed on selective LB agar medium containing antibiotics 50 mg/L kanamycin and 50 mg/L spectinomycin. After cultivation, the selected *E. coli* cells were transferred into a liquid medium LB containing 50 mg/l kanamycin and 50 mg/l spectinomycin. Isolation of plasmid DNA was carried out using a lysis method. Checking of insertion of the synthetic duplex into *E. coli* strain is performed by the restriction method digesting with *XhoI* and *XbaI* restriction enzymes (Fermentas, Thermo Fisher Scientific). The total restriction reaction mixture was 10 µl, including 4 µl of water, 4 µl of plasmid DNA (250 ng/µl), 1 µl of 10-fold restriction buffer, 0.1 µl of *XbaI* and *XhoI* restriction enzyme (1 unit each), mixed and incubated at 37°C for 16 hours, then digested plasmid was fractionated by 1.2% Agarose gel electrophoresis (Figure 2).

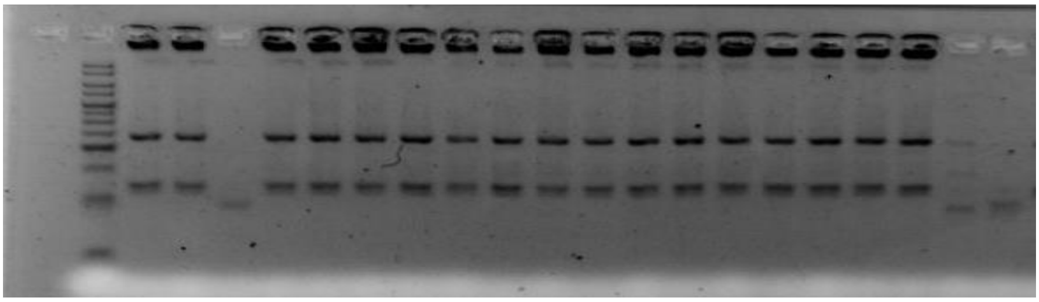

*Figure 2. Restriction reaction of inserted genetic construct into pART27 binar plasmid using Xho I and Xba I enzymes.*

The isolated and purified plasmid DNAs were transformed into *A. tumefaciens* cells (strain GV3101 and LB4404) and bacterium cells were grown on YEP medium (Bacto-peptone 10 g / l, sodium chloride 5 g / l, yeast extract 10 g/l, bacto agar 15 g/l, pH 7.2) containing antibiotics kanamycin (50 mg/l), gentomycin (40 mg/l) and spectinomycin (50 mg/l). Later, the selected cells for isolation of plasmid DNA were cultured in liquid YEB medium (Meat extract 5 g/l, yeast extract 1 g/l, peptone 5 g/l, sucrose 5 g/l, MgSO4- 493 mg/l, pH 7.2) containing kanamycin (50 mg/l), gentomycin (40 mg/l) and spectinomycin (50 mg/l).

The correct transformation of genetic construction into *A. tumefaciens* cells was checked by PCR amplification was implemented using promoter specific primer pairs. Subsequently, PCR products were purified, and electrophoresis was conducted in 2% agarose gel. Electrophoresis result then documented using Alpha Imager$^{TM}$ gel documented system (USA) (Figure 3).

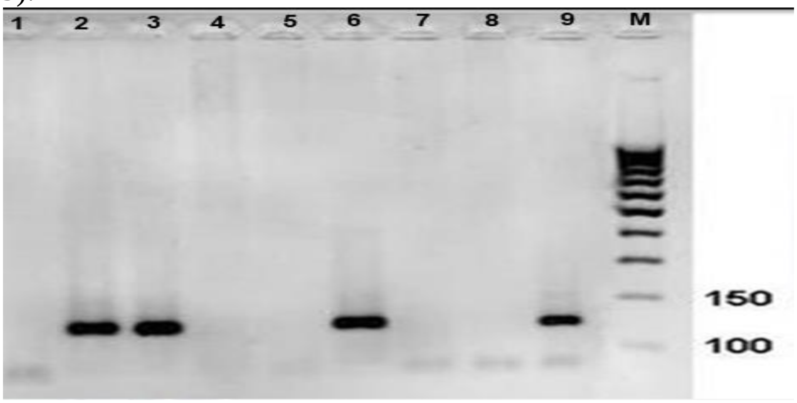


*Figure 3. Confirmation of transformation of genetic construction into A. tumefaciens by PCR amplification: 1-9 – the number of clones, M- marker, DNA Ladder 100.*

Firstly, targeted PCR products were cloned into pCR 2.1 TOPO-TA vector using TOPO TA Cloning Kit (Invitrogen) protocol. The ligation mixture (10 µl) was added into 100 µl of *E. coli* DH5α competent cells and incubated on ice for 30 min. After heat shock at 42°C in waterbath for 2 min, the cells were immediately transferred into ice and incubated for another 2 min. A SOC medium (0.5% (w/v) Bacto-yeast extract, 2% (w/v) Bacto-tryptone, 10 mM NaCl, 2.5 mM KCl, 10 mM $MgCl_2$, 10 mM $MgSO_4$ and 20 mM Glucose) of 500 µl was added, and the cells were incubated at 37°C with agitation (~220 rpm) for 45 min to 1 hr. The transformed cells were spread on LB/ampicillin (50 µg ml-1) /IPTG (0.5 mM)/ X-Gal (80 µg ml-1) plates and incubated at 37°C overnight and were screened for inserted PCR-products by digesting recombinant plasmids from clones with *EcoRI* restriction enzymes. Recombinant plasmids were isolated by miniprep kits (Invitrogen) for plasmid isolation and purification.

We used an optimized cotton tissue culture technique to transfer SynB RNAi construct into cotton tissue. Also used new modified medium in order to get somatic embryos from callus tissues. Detail information of used reagent content was decribed in Journal of Nature Communication. Transformation efficiency were determined by counting of RNAi embryos (Abdurakhmonov et al., 2014). As results we successfully obtained several synthetic RNAi plants based on cotton PHYB gene.

SynB RNAi T1 plants, taken from self-pollinated T0 generation seeds were planted into pots in phytotron condition in 2009 to check first morphological observations, such as number of flowers and bolls during flowering period. All data were scored weekly and monthly during vegetative and generative periods of plant growth, and then analyzed by One-way ANOVA, (JMP Genomics, SASTM and Sigma Plot programs). As results the number of flowers and bolls were significantly higher in SynB RNAi T1 compared to Cocker-312, (p-value 0.0127 for total flowers and 0.0201 for total bolls). T2 generation of SynB RNAi and control plants were then planted in the greenhouse to check stability of morphological changes. The percentage of seed germination, measurement of hypocotyl length, and first node distance parameter (HS) were recorded during vegetative period. Also number of flowers, bolls and opened bolls were counted weekly and monthly. However, no any significant differences between RNAi and control (Coker-312) were observed in seed germination and hypocotyl length.

Fiber samples from third generations of SynB RNAi lines were sent to “Sifat” center for comparative fiber parameters. For this reason, harvested samples were ginned to isolate seeds and lint. Fiber quality traits of RNAi and control lines, such as fiber length (upper higher mean-UHM) and strength (STR), micronaire (MIC), elongation (ELO), uniformity (UI), reflectance (Rd), and yellowness (+b) have been measured by HIV. The statistical analyses of fiber parameters of all generations of SynB RNAi, their hybrids and control lines were performed using the One-way ANOVA, (JMP Genomics, SASTM). The HVI result showed that fiber micronaire (Mic) of SynB RNAi T2-T6 lines were significantly lower than control lines, suggesting that RNAi treatment due to affect to fiber quality. This result was confirmed another evidence of our theoretical hypothesis. As results several fiber parameters were changed after RNAi treatment, and their stability also inherited to the number of generations.

**Conclusions**

For the first time, synthetic duplexes were synthesized based on cotton *PHYB* gene; for the first time, RNAi genetic construction have been developed based on synthetic duplex of *PHYB* gene; for the first time, A. thailana and *G. hirsutum* (var. Cocker-312) were transformed with pSynB RNAi vectors construct using in planta and in vitro tissue culture transformation techniques; for the first time, pSynB RNAi regenerate plants were obtained using somatically embryogenesis; The confirmation of the functionality of RNAi vector constructs in plant cells, and involvement in plant flowering and fiber micronaire have been shown by polymer chain reaction (PCR).

The practical results of the research are as follows: *PHYB* gene was cloned from *G. hirsutum* species. High variable “hinge” region was then determined from gene sequences. Synthetic oligonucleotide duplexes were created based on cotton *PHYB* gene. Genetic construction was designed into pART27 vector for synthetic oligonucleotide duplexes. Genetic construction (pSynB RNAi) and gene-knockout callus tissues (resistance to kanamycin antibiotics) were obtained. Gene-knockout embryos and regenerate plants were obtained based on callus tissues using somatically embryogenesis. Several generations of gene-knockout plants have been obtained, improved the main and lateral root system, early-flowering and fiber micronaire.

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## MOLECULAR PHYSIOLOGY AND BIOPHYSICS OF THE VOLUME-REGULATED ANION CHANNELS: VSOR AND Maxi-Cl

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Volume-regulated anion channels stay silent in the resting state. Whenever cells experience osmotic disbalance and swell, the channels open and provide a pathway for an efflux of the negatively charged osmolytes and osmotically obligated water to bring the volume back to normal. The volume-sensitive outwardly rectifying anion channel (VSOR) provides the major part of the swelling-induced anionic conductance at normal metabolic conditions whereas ischemic and hypoxic insults favor activation of the maxi-anion channel (Maxi-Cl). The VSOR has recently been identified as a hexameric complex of LRRC8 family proteins both in humns and mice. In order to identify the Maxi-Cl, we have isolated and fractionated proteins from membrane blebs with high channel activity and subjected to LC-MS/MS. Screening of the list of most likely candidates found in blebs by RNAi revealed SLCO2A1 protein as the molecular basis of the Maxi-Cl. Further proof of this hypothesis was obtained by gene silencing, gene knock-out by CRISPR/Cas9 technology, reconstitution of the purified recombinant protein and side-directed mutagenesis.

## NEW POLYPHENOL COMPOUNDS FROM *EUPHORBIA* PLANTS AND THEIR BIOLOGICAL ACTIVITIES

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Plants have been well documented for their medicinal uses for thousands of years. They have evolved and adapted over millions of years to withstand bacteria, insects, fungi and weather to produce unique, structurally diverse secondary metabolites. Natural products derived from plants such as polyphenols have received considerable attention in recent years due to diverse pharmacological properties, including anti-inflammatory, anti-parasitic, antioxidant, antiviral, anticancer and other activities. Due to their natural origins, the treatment products obtained from medicinal plants are of greater benefit in comparison to synthetic ones.

It is well known that the mitochondria are main units integrating signals that activate different ways of programmed cell death: apoptosis, autophagy, and necrosis like programmed cell death. Mitochondria are involved in a wide range of cellular processes of importance for cell regulation including the transport of different metabolites across inner mitochondrial membrane. In this case biologically active compounds demonstrating effect on mitochondrial processes are of a great interest and searching for a novel and effective safe drugs to prevent mitochondria dysfunction remains an area of intensive research.

This work focuses on the investigation of influence of three new polyphenol compounds, extracted from *Euphorbia* plants and encrypted as PC-1, PC-2 and PC-3, on functional parameters of mitochondria. It was shown that these polyphenol compounds have a positive effect to some functional parameters of mitochondria. For example, they inhibit the mitochondrial permeability transition pore (mPTP) opening, activates ATP-dependant potassium channel and has high antioxidant/antiradical activity.

Presently, epidemiological, biological and clinical investigations have demonstrated an important role of cell membrane oxidation in the progress of atherosclerosis, cataract and some forms of cancer caused by free radicals. Such natural antioxidants as vitamin C, E and β-carotene are significant factors for prophylactics of these diseases. Biological activity of many plant polyphenols can be considered as a result of their antioxidant activity (AOA). Here, we investigate the antioxidant (AOA) and antiradical activity (ARA) of PC-1, PC-2 and PC-3.

The effect of various concentrations of polyphenols on the process of lipid peroxidation (LPO) of mitochondrial membranes induced by the $Fe^{2+}$/ascorbate system *in vitro* experiments was studied. Adding of the $Fe^{2+}$/ascorbate system into the incubation medium induces LPO of mitochondrial membranes, resulting in violation of barrier function of the membranes, and the organelles swell sharply compared to the control. Under the induction of lipid peroxidation, polyphenols inhibit mitochondrial swelling at a concentration of 2 μM into the incubation medium, which indicates the antioxidant properties of studied compounds. The effect of PC-1, PC-2 and PC-3 on LPO in mitochondrial membranes depended on their concentration, i.e. with its increase in the incubation medium, the percentage of inhibition became more pronounced. Complete inhibition of liver mitochondrial swelling, i.e. of the LPO process was noted at a concentration of 10 μM of the tested compounds. At the same time, the concentration that caused the half-maximal inhibition of the LPO process ($IC_{50}$) for these polyphenol compounds were 6.08±0.06, 4,88±0,07 and 5,25±0,06 μM, respectively. Thus, in experiments it was shown that PC-1, PC-2 and PC-3 possess high AOA. Correlation coefficient between AOA and ARA of polyphenol compounds is **r=0.89**.

The mitochondrial permeability transition pore (MPTP) is now recognised to play a major role in both necrotic and apoptotic cell death. It is known that excessive ROS formation could promote the opening of mitochondrial permeability transition pore. Also, adding $Ca^{2+}$ ions into the incubation medium at a concentration of 10 μM causes mitochondrial swelling, that indicates the open state of mPTP. Experimental data show that polyphenol compounds from *Euphorbia* plants at a concentration interval of 10-100 μM in the presence of $Ca^{2+}$ ions inhibit the mPTP opening compared to the control.

Many scientists suppose that mito$K_{ATP}$ activation play an important role in ischemic diseases and have been reported to possess protective role against various cardiovascular complications. Several experimental studies gave shown a wide range of possible clinical use of mito$K_{ATP}$ openers. That's why next experiments were the investigation of mito$K_{ATP}$ activation by new polyphenol compounds PC-1, PC-2 and PC-3. Obtained data show that above mentioned compounds activate the mitoKATP channel.

Improving of functional characteristics of mitochondria by polyphenol compounds, served as the basis for the hypothesis of a possible antihypoxic effect of polyphenols due to its low toxicity. Considering that hypoxia is on the basis of various neurological disorders, in recent years many scientists think that the mitochondria are probable «targets» for organism adaptation to oxygen deficiency.

Therefore, the adaptive mechanisms that protect mitochondrial dysfunction during and after hypoxia, for instance, during hypoxic preconditioning, are effective molecular mechanism of action of compounds to improve of mitochondrial functional parameters.

## INORGANIC POLYPHOSPHATES IN PHYSIOLOGY AND PATHOPHYSIOLOGY OF MAMMALIAN CELLS

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Inorganic polyphosphates (polyP) are the big linear homopolymers, consisted from three to several hundred orthophosphate residues, linked together via high-energy phosphoanhydride bonds like in ATP molecule [1]. In the last 12 years, the functions of these polymers were widely studied in mammalian cells. It appeared that this so called "fossil" molecule play big role in cellular physiology and pathophysiology. It was found that inorganic polyphosphates participate in signal transduction in nervous system [1, 2], blood clotting [3], ion channels functioning [4-7], cellular bioenergetics [8], processes of protein folding as molecular chaperons [9-11] and in many other processes.

Our group for a long time studies the role of inorganic polyphosphates in mitochondria and their signalling properties in intracellular space.

On mitochondrial level, we had found that polyP affect mitochondrial ion transport. Application of inhibitors of different mitochondrial transporting systems had shown that polyP triggers the opening of mitochondrial permeability transition pore (PTP), without addition of external calcium. Based on the reports of other groups and our own results we concluded the inorganic polyphosphates are the essential natural triggers of PTP. Uncontrolled opening of PTP cause mitochondrial swelling and death, and if this process occurs in many mitochondria at the same time, it triggers cell death via apoptosis. Uncontrolled opening of PTP cause cell death during different ischemic pathologies and neurological diseases. Thus, the search for ways to regulate PTP is an important task for modern biology and medicine. Nowadays our group trying to find the connection between the level of polyP and cellular death caused by opening of PTP during ischemia reperfusion and Parkinson disease.

Inorganic polyphosphates can play role as intracellular signalling molecules. Our group had been found that polyP are the gliotransmitters and can transduce information among astrocytes. Experiments shown that polyP bind to the $P2Y_1$ purinoreceptors on the surface of astrocytes and launch the system of calcium signalling via activation of phospholipase C, which splits $PIP_2$ into $IP_3$ and diacylglycerol; $IP_3$ goes to the $IP_3$-receptors on endoplasmic reticulum and opens calcium channels and calcium begins to flow from the endoplasmic reticulum into the cytoplasm – which is a signal to different cellular processes. Recently, we found that inorganic polyphosphates trigger calcium signal in non-excitable cells as well via purinergic system.

Thus, polyP has a multiple action on cellular functions from mitochondrial calcium permeability and cell death to transducing the information between different cells via purinergic signalling system.

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# FACTORS CONTROLLING THE FORMING OF INTESTINAL FUNCTION SYSTEMS IN EARLY ONTOGENESIS

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The maturation of the digestive functions of the small intestine is determined depending on the ability of hydrolytic enzymes, such as lipases, proteases, amylases, glucosidases and lactases, to digest the appropriate substrates. Shifts in the enzymatic spectrum of the digestive tract also correspond to change in diet, especially during weaning.

Immediately after birth, the off spring of many immaturity mammals completely depend on the content of breast milk. At weaning breast milk with the main carbohydrate lactose is replaced by solid food, where carbohydrates are represented by starch, maltose, sucrose, and other poly- and disaccharides. During weaning the changes in the food composition correspond to changes in the activity of enzymes involved in the digestion of the respective substrates. During milk nutrition the activity of pancreatic α-amylase, γ-amylase, and maltase of the small intestinal mucosa is “pure” expressed, sucrase activity is absent and lactase activity is high expressed. By the time of transition to the definitive diet, sucrase activity appears, γ- and α-amylase and maltase activity significantly increases and the lactase activity sharply decreases. After a complete transition to self-feeding, a sharp increase in the activity of α-glucosidases with a simultaneous decrease in the activity of the brush bolder lactase is observed. The available data show that changes in the composition of food and diet play an important but not decisive role in the ontogenetic reorganization of the hydrolytic-transport systems of the digestive organs [1]. Changes in food composition play an important but not decisive role in the ontogenetic reorganization of hydrolytic and transport systems of the digestive organs during weaning.

Our data are shown participation of the adrenal and thyroid hormones well as insulin in the formation of intestinal function in early ontogenesis. Hydrocortisone has an inducing effect on the increasing in ontogenesis hydrolytic enzyme activity (α-amylase, maltase, saccrease, tregalase) without affecting declining in orthogenesis enzyme activity (brushbolder lactase). Thyroxin plays a decisive role in the development of the activity of those enzymes that decrease during individual development. The injection of thyroxin leads to an earlier repression of enteral lactase activity.

Experiment data show small intestine transport systems also are regulated by the hypothalamic-pituitary-corticoid and hypothalamic-pituitary-thyroid axes. The first one induces "definitive" transport systems (glucose absorption from maltose), without any effect on the transport of unbound glucose, which is well developed in the "pure" milk nutrition period. The second one causing a unidirectional effect on the transport glucose from maltose solution, leads to a premature decrease in juvenile unbound glucose intestinal transport. Insulin has a stimulating effect on both the hydrolytic-transport processes that develop and repress during ontogenesis, which are responsible for the assimilation of carbohydrates.

Adrenalectomy and thyroidectomy reduce the hydrolytic ability of the small intestine in relation to carbohydrates. Due to such functional reorganizations, the successful transfer of the offspring to a qualitatively new type of food is provided, and the lactating female is released from nursing and starts a new reproductive cycle.

Changing of the nursing mother hormonal status also affects the development of intestinal digestion in offspring. Adrenalectomy of the mother in the middle of the lactation period, stimulates the processes associated with the assimilation of definitive food, and does not affect the activity of "juvenile" lactase "in the offspring The daily injection of thyroxin to the lactating rat caused premature maturation of hydrolytic and transport system os small intestine in offspring. Pharmacological inhibition of the mother thyroid gland, on the contrary, delayed the development of hydrolytic and transport systems of the gastrointestinal tract in suckling rats. Chronic administration insulin to nursing mother stimulated the influence of cavity hydrolysis, membrane hydrolysis and absorption of carbohydrates.

Chronic intoxication with heavy metals (ions of lead, cadmium, mercury) reduces digestive capacity of the small intestine of growing organism. In the prevention and/or correction of heavy metal intoxication on the small intestine function among the tested biological substrates (flavonoids, terpenoids and non-starch polysaccharides) the most effective were non-starch polysaccharides (chitosan, inulin and pectin).

Strict analogy in the character and age dependence of small intestine enzyme and transport systems response to hormonal influences and environmental factors confirms the hypothesis of Ugolev [2], that hormone shifts are triggers for the transmission of external signals for the development of intestinal function in early ontogenesis. Our data show that such salts of the environment affecting the hydrolytic ability of the small intestine can be heavy metal salts.

It is known, the development of intestinal hydrolytic and transport systems in early ontogenesis is controlled by a genetic program, food composition, neurohumoral control, and environmental factors [3]. It is assumed that hormonal control is the main mediator in the genetic program realization under the influence of environmental factors in early postnatal ontogenesis in mammals.

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# AGROECOLOGICAL (RIVERS WATER, IRRIGATED LANDS) PROBLEMS OF THE UZBEKISTAN UNDER CLIMATE CHANGE

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**Abstract.** The report summarizes the results of authors many years of research by the authors on changes in the quality of water in the main rivers of the Syr Darya and Zarafshan (the Uzbek part) and problems of the salinization of irrigated lands on the example of the Navoi and Djizakh regions in the Uzbekistan.

There are three main sources of irrigation water in the Djizakh region with following average annual volumes of diverted water: from the Syrdarya river - 1890 mln $m^3$ (62.2%), from the Zarafshan River 1 050 mln $m^3$ (34.5%) and from the local Sangzor River - 100 mln $m^3$ (3.3%). Total water volume used in the province is 3 040 mln $m^3$, 2850 mln $m^3$ (93%) of which is used for agricultural purposes and remaining 190 mln $m^3$ (7%) is used by industry and drinking water supply.

The concentrations of organic matters in Syrdarya and Zarafshan rivers waters are very low, however they contain elevated concentrations of inorganic substances mainly in the form of chloride, sulfate and carbonate [1]. The water in midstream and especially downstream of ASB rivers including the Syrdarya and Zarafshan rivers has high mineralization (Table 1). This is mainly due to return of more than 55% of the volume of high-mineralized collector drainage waters from irrigated fields to the rivers [1-2].

*Table1. Long-term dynamics (2000-2016) of river water mineralization used for irrigation in Djizakh province (g/l)*

| Rivers | Hydro-posts | 2000 | | | 2005 | | | 2010 | | | 2017 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Minimum | Maximum | Average | Minimum | Maximum | Average | Minimum | Maximum | Average | Minimum | Maximum | Average |
| Syrdarya | Nadejdinskiy | 0.8 | 1.8 | 1.3 | 0.5 | 1.5 | 1.0 | 0.7 | 1.5 | 1.1 | 0.7 | 1.7 | 1.2 |
| Zarafshan | Pervomayskiy | 0.32 | 0.38 | 0.35 | 0.3 | 0.34 | 0.32 | 0.31 | 0.41 | 0.36 | 0.32 | 0.46 | 0.39 |

Most irrigated lands of the Republic of Uzbekistan are subject to salinity, due to the country's arid climate, and the geological and the hydrogeological conditions of its irrigated areas [1-3].

Based on results of conducted in detail analyses of long-term changes (1995 – 2017) in: the quality of water used for irrigation, the level and mineralization of GW, the ameliorative state of irrigated lands, as well as the salt balance in the system of "irrigated water - soil – drainage water" system with specific focus on Djizakh province it has been revealed that:

Gradual increase of crop yields (in particular 10-15% for cotton and grain) was achieved mainly due to the use of large volumes of mineral fertilizers. However, excess amounts of mineral fertilizers (primarily nitrogen containing) can also cause salinization of soils.

The GW table depends on the relief of the irrigated areas, the depth of drainage, the proximity or range distance from the surface and underground streams. High salinization level of groundwater can lead to increase of salts on upper layers of the soil through capillaries, which affects the root zone of crops, reducing their productivity. The best way to effective control the irrigated lands salinization is to reduce the table of GW, to increase the efficiency of drainage networks that divert drainage waters and most importantly, to use modern climate smart water saving irrigation techniques.

The analyses of monitoring results have clearly demonstrated that the ameliorative condition of the irrigated lands of the province has not improved significantly. The "irrigation water - GW – soil" meteorological system is extremely complex, which changes permanently over time and requires monitoring and analysis of a large number of parameters.

The obtained scientific and practical results (on example of Jizzakh province irrigation zone) based on long-term monitoring data have revealed that optimal use of the available regional water resources, improvement of reclamation status of irrigated lands, maintaining GW layer and mineralization as well as irrigated soils salinization at an optimal level are extremely important to achieve sustainable use of water and land resources under changing climate.

**Our recommendations:**

- Increasing the density and depth of drainage system for efficient use of irrigated lands and protection against salinization;
- Implementation of laser leveling with flatteners after completion of plowing operations;
- Carrying out reclamation leaching works timely and with high quality;
- Rational use of irrigational water by farmers, decreasing diversion of sewage to collectors and maintenance of technical parameters of collectors;
- Leaching of soils elevated salinity requires using additional water resources, which increases groundwater table, mineralization and accelerate salinization process of irrigated soils.
- Repeatedly conducted saline soils leaching on irrigated lands reduces surface soil salinity, but due to the fact that current drainage systems cover less than the irrigated land areas in combination with their ineffective operation can lead to permanent salinization, fertility decline and reduced yields of agricultural crops.
- The use of modern GIS technologies combined with experimental methods in irrigated lands monitoring process: GW table, salinization of water and soils, "irrigated water - soil – drainage water" salt balance, etc. will give good results.

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# AUTHENTIC MATERIALS IN ENGLISH LISTENING CLASS

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The important factor in teaching foreign languages is students' motivation. Due to the rocketing development of innovative technology, nowadays many regular textbooks are getting outdated, so it must be the teacher's priority to implement study materials which will be up to date, interactive and engaging to meet the learning targets. Authentic materials can be beneficial to the learning process as long as it is properly used (Guariento & Morley, 2001)

What do we mean by authentic materials?

It should be noted that there are many definitions but I will stick to that of Morrow: "An authentic text is a stretch of real language, produced by a real speaker or writer for a real audience and designed to convey a real message of some sort." (Morrow 1977:13)

Who are authentic materials suitable for?

Studies show that they can be the most beneficial to students beginning from intermediate, because beginners and elementary students fid original texts less encouraging and even lowering the students' self-esteem and consequently inner motivation (Peacock 1997)

Why choose authentic materials?

Let's have a look at some of their advantages.

- Authentic materials bring learners into direct contact with a reality level of Business English.

*Real business English* – that is used by business people to communicate with other business people – English that hasn't been made especially easy for learners – can be motivator. The other extremely important point is that many learners are already in business so they will have had exposure to the English language that is used to conduct *real business*.

- Authentic materials drawn from periodicals are always up-to-date.

English itself is constantly developing and changing, so working with up-to-date materials not only means that the content of the material is up-to-date. It is also part of the work of business people to be aware of current news issues.

There are majority types of study materials among which I would like to mention TED presentations (recorded videos). TED (Technology, Entertainment, Design) is non-profit organization which has been publishing various speeches and conference reports online since 2006. TED conferences and congresses have been held annually since 1990, and the topics of speeches are inexhaustible. The talks cover pressing issues and innovative discoveries in technological development, design, society, culture, education, human rights etc. As of March 2016, the website of the conference www.ted.com features more than 2400 public speeches. The average length of a talk is 18minutes, and among the speakers, you can find renowned contemporaries, such as David Cameron, Bill Gates and many others. All these features combined make TED presentations highly suitable for teaching listening and communication skills, with regard to modern learners' need and study text requirements (Field 2002).

Among the pros of using TED presentations as study language material the following is pointed:

1. Usability:

1.1 The majority of talks are in English, with subtitles in more than 40 languages, and there is often an interactive typescript.

1.2. All TED recordings are available online as video and audio presentation to download. Furthermore, there is an application for most smartphone models, which is intelligible and user-friendly. In addition, mobile-friendly interface has been noted to empower learner autonomy in acquiring listening skills, encouraging the students to work out of class (Reinders & Cho 2010).

2. Content:

2.1. Given the number of conference participants, it is possible to raise students' awareness of different accents and English variations. There are native and non-native speakers, all communicating fluently and successfully in English. While being demonstrative, it is also encouraging, setting the example of outstanding language skills even in non-native speakers.

2.2. There is a huge variety of subjects and topics, making the talks perfect for both teaching English for specific academic purposes and general cultural and social awareness.

2.3. TED-talks are modern, well-structured speeches which are often accompanied by visual aids, such as graphs, pictures, tables, and charts and video clips. They can be used in preparing the students for delivering their own presentations and speeches, with focus on presentation techniques and skills.

As for the tasks, their number is evidently very vast, although they must be chosen according to students' level of proficiency of English and their communicative and comprehensive skills.

# BIOLOGY AND MEDICINE

## Short Communications

### TOLERANCE OF GENETICALLY DISTANT COTTON HYBRIDS TO BOLLWORM IN DEPENDING OF THE LEVEL GOSSYPOL IN SEEDS

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**Introduction.** Cottonseed provides a high quality protein that is currently under utilized because of the presence of a toxic compound called gossypol. Gossypol is biosynthesized by the free radical coupling of two molecules of hemigossypol. During this coupling reaction, two optically active enantiomers are formed. One of these is referred to as (+)-gossypol and the other as (-)-gossypol. In cotton (*Gossypium* L.) the ratio of (+)- to (-)-gossypol can vary from 98:2 to 31:69 in seed. Within the genus *Gossypium*, were found accessions from several species that have >92% (+)-gossypol in the seed. These include *G. mustelinum*, *G. anomalum*, and G. *gossypioides* first reported that *G. barbadense* had an excess of the (-)-enantiomer in the seed. It was found that some accessions of *G. darwinii*, *G. sturtianum, G. harknessii*, *G. longicalyx* and *G. costulatum* also produce an excess of (-)-gossypol in the seed. [1]

Gossypol, which is found in pigment glands, was identified in 1915 as the toxic component in cottonseed. Gossypol was toxic to cotton aphids, lygus bugs, salt-marsh caterpillars, thurberia weevils, and bollworms. The grape colaspis and leaf beetle preferred feeding on glandless compared to glanded cotton cultivars. Gossypol inhibits the growth and development of many insect pests including the beet armyworm, bollworm, cabbage looper and the salt-marsh caterpillar.[2]

Terpenoids that are biosynthetically related to gossypol also occur in the foliage. Besides gossypol, these terpenes include hemigossypolone and heliocides $H_1$, $H_2$, $H_3$ and $H_4$. These compounds are also involved in insect resistance. In an artificial diet study, Stipanovic and collaborators established the effective dosage that is required to reduce growth of the tobacco budworm larvae (*Heliothis virescens*) by 50% ($ED_{50}$). Although gossypol is toxic, field studies show that the levels of heliocides and hemigossypolone correlate better with resistance than gossypol. Cotton bollworm (*Helicoverpa armigera*), the main rodent pests. In particular, the elements of the cotton bollworm moth is a great loss, he is the glory and the loss of knots, dry or completely fall. Carried out at the United States that the resistance of cotton bollworm and sucking pests in addition to the gossypol, along with other terpenoid compounds, flavonoids, fatty acids and tannin material is provided. [3]

**Materials and methods**. By joint researches of under "KX-EA-KX-2018-65" project toward developing of cultivars that exhibit a high ratio of (+)-gossypol in the seed by using of Uzbek cultivars and American lines has shown that: -it is possibility to transfer the high (+)-gossypol seed trait from U.S. cotton accessions into Uzbek cultivars; these Uzbek cotton hybrids developed to date have >93% (+)-gossypol; thus, it is possible to introduce the high (+)-gossypol seed trait into Uzbek cotton lines to provide plants with agronomic traits suitable for growing in Uzbekistan; these plants are no more susceptible to insect pests and pathogens than normal cotton cultivars.

**Results and discussion.** Based on the above researches, we will analyse in this paper some research results regarding to *Helicoverpa armigera* resistance of progenies developed with the participation of ecologically-geographically and genetically distant hybridization.

The results of first observation of hybrids $F_6$ under the conditions of a greenhouse, showed that all combinations with both a high (above 90%) and low (below 70%) level of (+) - gossypol were affected by *Helicoverpa armigera*. Among the studied, the hybrid $F_6BC_3S_1$-1-6-3-15 x S-6530 with a relatively high content of (+) - gossypol (90.8%), was less affected (6%). The results of monitoring of susceptibility to *Helicoverpa armigera* at the first observation showed that the pest is populated depending on the level of (+)-gossypol. In other words, among $F_6$ hybrids with a high level of (+) -gossypol, only in two cases, i.e. at $F_6$L-10/04 x $BC_3S_1$-47-8-1-17 and $F_6BC_3S_1$-1-6-3-15 x S-6524, a high degree of sussceptibility were observed (23% and 21%, respectively). It can be noted that hybrids of this generation with a high level of (+) - gossypol were affected by *Helicoverpa armigera* from 6% ($F_6BC_3S_1$-1-6-3-15 x S-6530) up to 17% ($F_6BC_3S_1$-47-8-1-17 x S-6532). Thus, it was found that the studied progenies $F_6$ with high (+)-gossypol were relatively resistant to the cotton bollworm in comparing to progenies with low level (+)-gossypol. These results indicate that the level of (+) - gossypol does not significantly affect on resistance to *Helicoverpa armigera*.

The results of studies of another group of hybrids developed between US accessions and Uzbek cultivars with a high level of (+)-gossypol in seeds showed that their affection related to the initial form genotype involved in hybridization. For example, such hybrids with a low level of (+)-gossypol in seeds as $F_3$Bukhoro-8 x $BC_3S_1$-1-6-3-15 (62%), $F_3$Turon x $BC_3S_1$-1-6-3-15 (67%), $F_3$Surkhon -14 x $BC_3S_1$-1-6-3-15 (79%) and $F_3$I-9871 x $BC_3S_1$-1-6-3-15 (75%), developed from a relatively resistant parent form ($BC_3S_1$-1-6-3-15) with high (+)- gossypol in seeds (93.8%), were affected with *Helicoverpa armigera* in less degree (with respective affection 25%, 50%, 40% and 40%).

Among the hybrids $F_5$ with a high level of (+) - gossypol in the seeds, only in one case the damage was 50% ($F_5BC_3S_1$-1-6-3-15 x S-6530), and in the remaining cases susceptibility were 55- 65%. In contrast to the above, hybrids of this generation with a low level of (+) - gossypol in seeds, differed in relative tolerance to the *Helicoverpas armigera*. For example, susceptibility of $F_5BC_3S_1$-47-8-1-17 x S-6532 and $F_5BC_3S_1$-1-6-3-15 x S-6532 - with low (+)-gossypol were 15% and 20%, respectively. The remaining hybrids of this generation with a low level of (+)-gossypol were affected by a *Helicoverpa armigera* from 30% up to 45%.

Among the hybrids $F_6$, comparative resistance with a low level of (+) - gossypol and a relatively high affection of buds with a high level of (+)-gossypol (over 90%) are also observed. For example, the damage of buds with a low (+)-gossypol was from 30% ($F_6BC_3S_1$-1-6-3-15 x S-6524) up to 40% ($F_6BC_3S_1$-1-6-3-15 x S-6532 and $F_6$L-10/04x $BC_3S_1$-47-8-1-17), and with a high level of (+) - gossypol - from 55% ($F_6BC_3S_1$-47-8-1-17 x S-6530 and $F_6BC_3S_1$-1-6-3-15 x S-6524) up to 70% ($F_6BC_3S_1$-47-8-1-17 x S-6532). The similar dates of damaging of buds with different levels of (+) - gossypol were observed at hybrids $F_7$.

**Conclusion**. Thus, on the basis of the obtained results of the study of comparative resistance to *Helicoverpa armigera* among hybrids of different generations in petri dishes, it was established that progenies with a different levels of (+)-gossypol, a definite pattern is observed for affection. Genotypes with a low level of (+)- gossypol are affected by *Helicoverpa armigera* to a certain extent less than hybrids with high (+)-gossypol level. Although, the incidence of the initial accession $BC_3S_1$-1-6-3-15 with a high level of gossypol does not preclude the possibility of developing of resistant genotypes with a high level of (+)-gossypol, which requires additional studies in this direction.

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# SPLINE METHODS IN BIOMEDICAL SIGNALS PROCESSING

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Biomedical - one of the most popular areas of signal processing applications, especially in the last decade. Most of the physiological processes in a living organism is accompanied by the appearance of signals. Diseases or defects in the body lead to pathological processes, i.e. to changes in normal physiological processes. As a result of the presence of a particular pathology signals may differ from normal signals and used to diagnose diseases.

In the field of biomedical signals processing, one often encounters a problem in which, according to the experimental data, it is necessary to restore the general character of the phenomenon or process. One of the ways to solve this problem is to use spline methods for the approximation of experimental data.

The main properties of B-spline functions offer the possibility to implement algorithms of interpolation in a faster and optimal manner. A function can be represented by B-spline functions with a set of coefficients. For interpolative signal reconstruction it is necessary to calculate those coefficients. In this paper, for cubic spline interpolation it is analyzed a known algorithm and some of his deficiencies. Also there are relieved some possibilities for developing new algorithms that could eliminate those problems. It is presented another way to determine the initial coefficients by using the polynomial representation on short intervals of the spline function and his derivatives. Based on this results are made several observations for further use in improving the algorithm and memory usage.

The wide popularity of spline methods is explained by the fact that they serve as a universal tool for the approximation of functions and, in comparison with other mathematical methods, while the information and hardware equals them provide greater accuracy.

Any spline $S_m(x)$ of level $m$ of defect 1, that interpolates the given function $f(x)$ may be represented by an only way by B-splines as[1]:

$$f(x) \cong S_m(x) = \sum_{i=-1}^{m+1} b_i \cdot B_i(x), \quad a \le x \le b, \tag{1}$$

where $b_i$ – coefficients, and $m$ – level of Spline.

The computational problems are greatly alleviated by turning to a local spline approximation in which the values of the approximating spline function at each cut depend only on the values of the function being calibrated from some locality of this separation. Another feature of these methods is that they do not require solutions of systems of algebraic equations when the parameters of the spline are nested. The required amount of computing work does not depend on the number of grid nodes, but is determined only by the degree of the spline.

The function value is calculated using the formula[1]

$$f(x) \cong S_3(x) = b_{-1}B_{-1}(x) + b_0B_0(x) + b_1B_1(x) + b_2B_2(x) \quad for \; x \in [0,1] \quad (2)$$

The rest of basic splines at this sub-interval are equal to zero and, consequently, do not participate in formation of the sum (2).

Different methods exist for calculation of b-coefficients: interpolation and “points” formulae, smoothing splines and the smallest quadrates method. However, the “points” formulae should be used for the systems that function in real time.

A parallel-pipeline computation structure has been developed for implementation of one-dimensional basic spline-approximation. It allows saving the memory for storage of values of basic splines twice at a limited number of processors(Figure 1).

A spline-method of analysis, processing and determination of signals of biomedical structures, based upon the use of basic signals are developing. The developed method allows assessing the fatigue and prognosticating the stability and strength of biomedical signals.

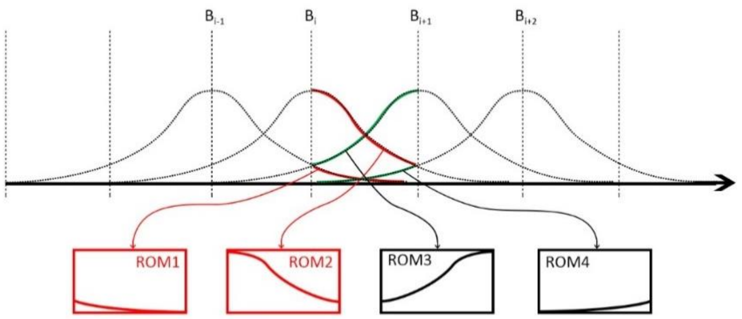


*Figure 1. Main structure of restoration bicubic splines in a memory.*

The work of the software. F(x) is given gastro signals and S(x) is array of values of the restored function based on spline methods. All programs are interrelated via the program complex interface and they work interdependently with each other (Figure 2.).

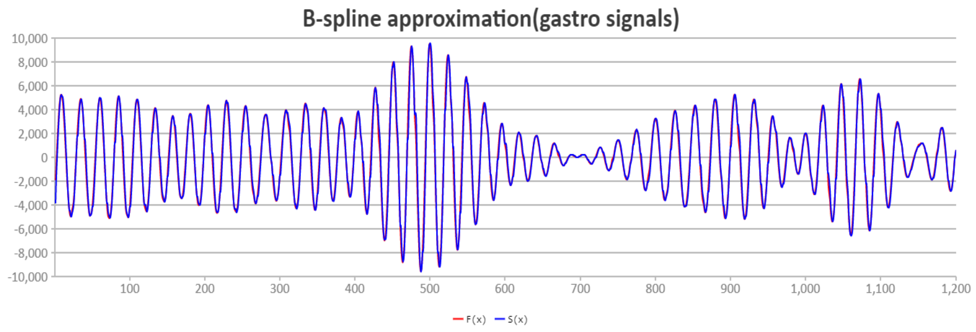


*Figure 2. The program complex interface with a result.*

The advantages of spline methods are the simplicity of description and greater proximity to the practical requirements of computing mathematics. Mainly, the following two reasons made it possible for rapid development of the one variable spline-functions theory as a tool of digital analysis:

- Good convergence of splines with the approximation objects;
- Simplicity in implementation of algorithms of constructing splines on computers;
- Reliable and fast computation on real time systems;

As opposed to polynomials, no particular difficulties occur with the issue of existence and uniqueness of solution in interpolation of functions of many variables for splines.

The results will be described in digital processing of biomedical signals based on Spline methods in presentation.

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## GEOPHYSICAL SIGNALS PROCESSING ON THE BASIS OF BICUBIC SPLINE FUNCTION


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The use of interpolative cubic splines in the processing and recovery of various geophysical and other types of signals, and the use of bicubic interpolation spatial functions in the construction of a mathematical model are currently relevant in the development of science and technology.

$S_n$ (f; x) function is local interpolation spline function on n-level, if following conditions are met:

$S_n$ (f; x) ϵ $H_n[x_i, x_{i+1}]$, 2) $S_n$ (x) ϵ $C^m[a,b]$, 3) $S_n$ $(x_i)$ = f(x) i=0,n.

the n-level interpolation spline function defines the n = n - m defect number.

Two variable local interpolation bicubic spline functions are constructed based on a variable local interpolation cubic spline function. The errors are evaluated on the basis of the error of the local interpolation cubic spline function, as follows. $S_{3,3}(x,y)$ is a local bicubic spline function, to interpolate $f(x,y)$ function, in the $f_{ij}$ knot values $[x_i, x_{i+1}]\times[y_j, y_{j+1}]$ are as follows;

$$(x_{i-1}, y_{j-1}),\ (x_{i-1}, y_j),\ (x_{i-1}, y_{j+1}),\ (x_{i-1}, y_{j+2}),$$
$$(x_i, y_{j-1}),\ (x_i, y_j),\ (x_i, y_{j+1}),\ (x_i, y_{j+2}),$$
$$(x_{i+1}, y_{j-1}),\ (x_{i+1}, y_j),\ (x_{i+1}, y_{j+1}),\ (x_{i+1}, y_{j+2}),$$

$$(x_{i+2}, y_{j-1}),\ \ (x_{i+2}, y_j),\ \ (x_{i+2}, y_{j+1}),\ \ (x_{i+2}, y_{j+2}).$$

It should be noted that One-dimensional local cubic Spline is function which is spline for a constant value of one of the variables for the variable than other variable.
x will be fixed, that is, the local cubic spline has the following shape:

$$S_3(x_i, y) = (1-u)Z_j(x_i, y) + uZ_{j+1}(x_i, y), \tag{1}$$

In here

$$Z_j(x_i, y) = -\frac{1}{2}u(1-u)f_{i,j-1} + (1-u^2)f_{ij} + \frac{1}{2}u(1+u)f_{i,j+1}, \tag{2}$$

$$Z_{j+1}(x_i, y) = \frac{1}{2}(1-u)(2-u)f_{ij} + u(2-u)f_{i,j+1} - \frac{1}{2}u(1-u)f_{i,j+2} \tag{3}$$

Accordingly, Parabolas cross the point

$$(x_i, y_{j-1}),\ (x_i, y_j),\ (x_i, y_{j+1});\ \ (x_i, y_j),\ (x_i, y_{j+1}),\ (x_i, y_{j+2}),$$

$$u = \frac{y - y_j}{l},\ \ l = y_{j+1} - y_j.$$

$$S_3(x_i, y) = -\frac{1}{2}u(1-u)^2 f_{i,j-1} + \frac{1}{2}(1-u)(2+2u-3u^2)f_{ij} +$$

$$+\frac{1}{2}u(1+4u-3u^2)f_{i,j+1} - \frac{1}{2}u^2(1-u)f_{i,j+2}, \tag{4}$$

$j = \overline{0, M-1},\ 0 \le u \le 1.$

Likewise, the following dimensional splines are also taken

$$S_3(x_{i-1}, y) = (1-u)Z_j(x_{i-1}, y) + uZ_{j+1}(x_{i-1}, y), \tag{5}$$

$$S_3(x_{i+1}, y) = (1-u)Z_j(x_{i+1}, y) + uZ_{j+1}(x_{i+1}, y), \tag{6}$$

$$S_3(x_{i+2}, y) = (1-u)Z_j(x_{i+2}, y) + uZ_{j+1}(x_{i+2}, y), \tag{7}$$

Followed by, constants will be fixed $x = x_{i-1};\ \ x_{i+1};\ \ x_{i+2}$.

Once again using the cubic spline construction, after (3), (4) are simplified, the bicubic spline function is constructed and appears as following shape:

$$S_{3,3}(x,y) = \varphi_1(t)\left[\varphi_1(u)f_{i-1,j-1} + \varphi_2(u)f_{i-1,j} + \varphi_3(u)f_{i-1,j+1} + \varphi_4(u)f_{i-1,j+2}\right] +$$

$$+\varphi_2(t)\left[\varphi_1(u)f_{i,j-1} + \varphi_2(u)f_{i,j} + \varphi_3(u)f_{i,j+1} + \varphi_4(u)f_{i,j+2}\right] +$$

$$+\varphi_3(t)\left[\varphi_1(u)f_{i+1,j-1} + \varphi_2(u)f_{i+1,j} + \varphi_3(u)f_{i+1,j+1} + \varphi_4(u)f_{i+1,j+2}\right] +$$

$$+\varphi_4(t)\left[\varphi_1(u)f_{i+2,j-1} + \varphi_2(u)f_{i+2,j} + \varphi_3(u)f_{i+2,j+1} + \varphi_4(u)f_{i+2,j+2}\right]. \tag{8}$$

In here $i = \overline{0, N-1},\ j = \overline{0, M-1},\ 0 \le t \le 1,\ 0 \le u \le 1,\ t = \frac{x - x_i}{h},\ u = \frac{y - y_j}{l},\ h = x_{i+1} - x_i$

$$, l = y_{j+1} - y_j,\ \varphi_1(t) = -\frac{1}{2}t(1-t)^2,\ \ \varphi_2(t) = \frac{1}{2}(1-t)(2+2t-3t^2),$$

$$\varphi_3(t) = \frac{1}{2}t(1+4t-3t^2),\ \ \varphi_4(t) = -\frac{1}{2}t^2(1-t)$$

Nowadays, Reconstruction and identification of geophysical signals is one of the most important issues (Figure 1):

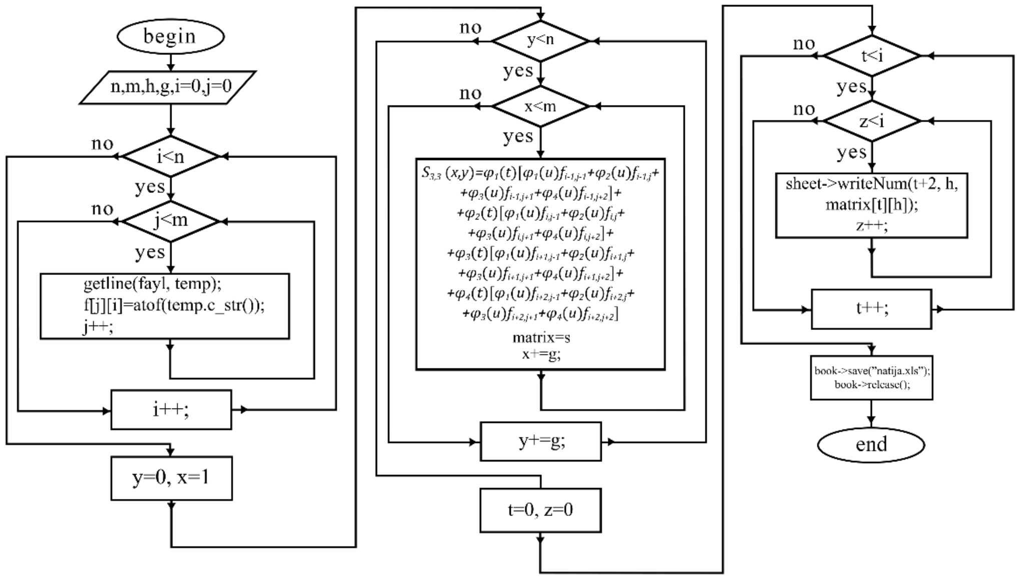


*Figure 1. Block scheme of application of two-dimensional geophysical signals processing by bicubic splines.*

Modeling two-dimensional geophysical field was performed by using this model. The results of the modeling will be displayed in the presentation.

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## BIOMEDICAL SIGNALS INTERPOLATION CUBIC SPLAIN MODELS


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It is not always easy to find solutions to science and technology through a specific method. In many cases, the problem is that the problem can be greatly enhanced by the use of advanced techniques for finding approximate solutions based on predetermined conditions, as well as the creation and implementation of mathematical models, and in increasing reliability of science and technology.Including in the field of radio electronics, transportation, aviation, medical, economics and others.

Building biomedical signals for the diagnosis of diseases in the medical field is one of the most important issues today[1].

In this article, preliminary experimental data was obtained for the diagnostic analysis of gastroenterological diseases and based on this information, Lagrangian interpolation model and local interpolation cubic spline models were constructed.

The construction of the Lagrangian interpolation model is based on the idea of replacing functions with a function that is relatively close and structurally simple.
Application of the Lagrangian interpolation model in digital processing of signals is more convenient when nodes are low. However, there are many inconveniences because the signals are real numbers and their values are high.

This leads to the construction of a high level of the polinominal, and secondly, the coefficients of multiplier are of great value and the error of the interpolation through the model is high. The error can also be seen when a graphical view is rendered.

This is easily understood through spline features. The Lagrangian interpolation model and the local interpolation cubic spline models are used for digital processing of gastroenterological signals to differentiate these defects.

Construction of models (Table 1) is based on the preliminary data of the gastroenterological signal.

Table 1. preliminary data of gastro signal.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| X | 0.5 | 1 | 1.5 | 2 | 2.5 | 3 | 3.5 | 4 | 4.5 |
| Y | 0.051 | 0.056 | 0.049 | 0.069 | 0.097 | 0.132 | 0.066 | 0.118 | 0.080 |
| X | 5 | 5.5 | 6 | 6.5 | 7 | 7.5 | 8 | 8.5 | 9 |
| Y | 0.090 | 0.072 | 0.043 | 0.112 | 0.128 | 0.109 | 0.056 | 0.121 | 0.091 |
| X | 9.5 | 10 | 10.5 | 11 | 11.5 | 12 | 12.5 | 13 | 13.5 |
| Y | 0.138 | 0.142 | 0.119 | 0.053 | 0.074 | 0.107 | 0.107 | 0.085 | 0.058 |
| X | 14 | 14.5 | 15 | 15.5 | 16 | 16.5 | 17 | 17.5 | 18 |
| Y | 0.021 | 0.004 | 0.037 | 0.036 | 0.099 | 0.094 | 0.075 | 0.064 | 0.034 |
| X | 18.5 | 19 | 19.5 | 20 | 20.5 | 21 | 21.5 | 22 | 22.5 |
| Y | 0.067 | 0.045 | 0.063 | 0.069 | 0.069 | 0.097 | 0.077 | 0.066 | 0.093 |
| X | 23 | 23.5 | 24 | 24.5 | 25 | 25.5 | 26 | 26.5 | 27 |
| Y | 0.061 | 0.048 | 0.085 | 0.113 | 0.078 | 0.051 | 0.056 | 0.049 | 0.069 |

The Lagrangian classical interpolation model was built on the subject of interpolation of gastroenterological signals.

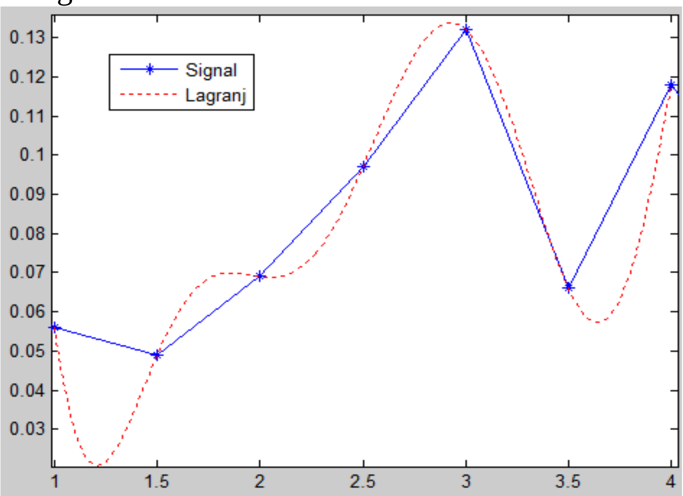


*Figure 1. Lagrangian interpolation of the gastroenterological signal.*

Local interpolyation cubic spline model was constructed using gastroenterological alarm values and graphic views were created with the Lagrangian interpolation model (Fig.2).

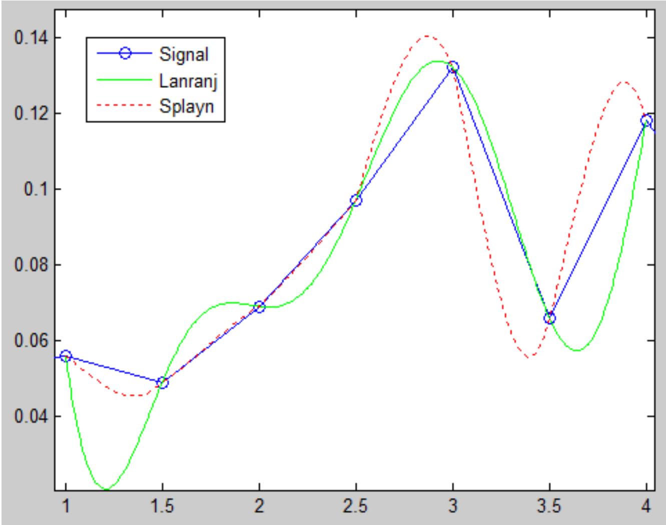

*Figure 2. Interpretation of gastroenterological signal.*

In summary, interpolation is crucial in digital processing of signals. In this study, Lagrangian classical interpolation and local interpolation cubic spline models were constructed and analyzed. As a result of the analysis, the spline properties showed that the interpolation of signals is better than interpolation through classical polynomials. Thus, the use of spline features in digital processing of signals will be effective.

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## IMPLANTATION OF INFORMATION TECHNOLOGIES IN REHABILITATION

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In recent years, the state social program in Uzbekistan related to persons with disabilities has been mainly social support: the allocation of social benefits, the establishment of various benefits for housing, utilities, study, travel, free provision of medicines, rehabilitation tools, technical means of rehabilitation, etc. All these have a positive effect on improving the lifestyle of people living with people with disabilities. But their treatment, recovery and quality rehabilitation issues remain relevant.

One of the ways to solve these problems is the introduction of information and Internet technologies into the system of vocational rehabilitation of the disabled, which is a recovery process.

Information technology (IT) in the modern world are used everywhere. Health is no exception. Modern IT developments have a positive impact on the development of new ways of organizing medical care for the population. A large number of countries have long been actively using new technologies in the health sector. Teleconsultations of patients and staff, the exchange of information about patients between different institutions, remote recording of physiological parameters, monitoring of operations in real time — all these possibilities are provided by the introduction of information technology into medicine. This leads informatization of healthcare to a new level of development, having a positive effect on all aspects of its activity. IT in medicine provide an opportunity to conduct high-quality monitoring of patients.

Disability can be in the motor organ, speech organ, auditory organ, vision organ and other internal organs of a person.

By type of disability, there are various means of rehabilitation based on computer technologies and the number of special computer programs developed by various companies and scientific centers and the level of equipment of rehabilitation centers with computer equipment and the number of people using it are constantly growing.

The rehabilitation of people with disabilities requires an individually-integrated approach that improves the effectiveness of treatment and rehabilitation. Analyzes showing existing systems and tools does not allow for an integrated approach in its functional limitations. For example, the rehabilitation of people who have suffered a stroke requires social rehabilitation, which consists of several stages requiring special hardware and software tools of computer technology.

At the Karshi branch of Tashkent University of Information Technologies named after Muhammad al-Khwarizmi a new scientific laboratory “Measuring and Computing Complex” was organized and equipped with modern special computer equipment under the TechReh project under the Erasmus + program. This system is widely used in France, Italy, Portugal and other European countries. In Uzbekistan, the computer system “Measuring and Computing Complex” is little known.

The laboratory complex consists of six modules:

- module for the study of muscle activity;
- G-sensor of kinematic measurements in the space X, Y, Z;
- optoelectronic system;
- power platform;
- module for rehabilitation by immersion in virtual reality;
- hardware and software for the treatment of patients with aphasia.

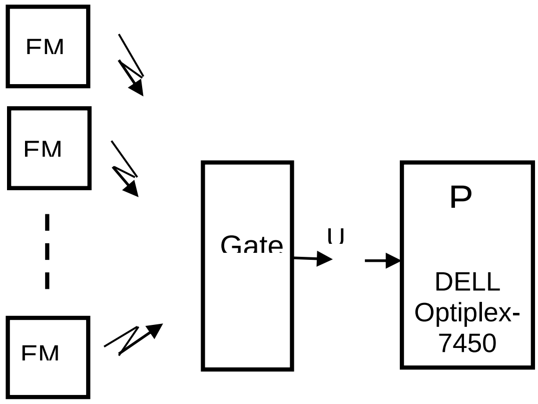

*Figure1. Module for the study of muscle activity*

Where EMG is a muscular electromyograph. To transmit information recorded by muscle electromyography is transmitted to the receiver wirelessly using ZIGBEE technology (f = 2.4 GHz).

Rehabilitation of stroke patients is a complex and lengthy process, which includes not only and not so much medical rehabilitation as social. The use in the rehabilitation process of information technologies, and in particular, the computer system “Measuring and Computing Complex”, reduces the patient's feeling of helplessness, lack of demand, nervousness, and allows to solve communication problems. As a result, the motivational and volitional component of the rehabilitation process and the effectiveness of stabilizing the personal status of people who have had a stroke increase.

In addition, using the capabilities of these systems, it is possible to conduct a study on the rehabilitation of children with the problem of motor or speech systems, assessing the condition of healthy athletes, which is effectively used to evaluate healthy football players of Italian football clubs, research and development of new means of rehabilitation, research of muscle activity, research of digital algorithms processing of bio-signals, the study of methods of statistical processing and the creation of intelligent systems.

This article was prepared based on the results of a study using special medical equipment obtained ba European Union grant in the framework of the TechRehab Erasmus + project 561621-EPP-1-201-IT-EPPKA2-CBHE-JP in the Karshi branch of TUIT.

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## SCREENING FOR EXOPOLYSACCHARIDE SYNTHESIS IN SEVERAL LACTOBACILLUS STRAINS ISOLATED FROM LOCAL PLANTS

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Exopolysaccharides (EPS) are high molecular weight and biodegradable polymers and synthesized by a wide range of bacteria, including Lactic Acid Bacteria (LAB) strains [1]. LAB strains are widely used in medicine and traditional dairy products for their ability to normalize the intestinal microbiote and wide range health benefits. Recent studies on LAB exopolisaccharides have shown beneficial health effects of such exopolisaccharides and thus interest for LAB produced exopolysaccarides has been increased [2]. LAB exopolisaccharides are divided in two groups homo- and hetero-polysaccharides according to their structure [3]. LAB strains synthesize polisaccharides using various biosynthetic mechanisms in regard to construct components of cell wall [4] and the synthesis procedure occurs by utilizing energy from ATF [5].

*Lactobacillus plantarum* Mal, *Lactobacillus plantarum* K-2, *Lactobacillus casei* P-1, *Lactobacillus spp.* TM-5, *Lactobacillus agilis* Val strains were screened for exopolisaccharide production in this study. The strains were isolated from local grown plants and identified at the Institute of Microbiology, Uzbek academy of Sciences. The strains were cultured in 10 ml MRS Broth (MRS Broth for cultivation Lactobacillus, 0.2% carbon source, TM Media) to OD=0.2 and subsequently were inoculated into 1000 ml MRS Broth (pH=6.7) at 0.5% v/v, respectively. Then incubated for 48 h at 37ºC without agitation and after incubation the each culture was centrifuged at 7000 ×g for 20 minutes at 4ºC to remove bacterial cells. Protein content was precipitated using 14% trichloroacetic acid and removed by centrifugation and two fold absolute cold ethanol by volume was added to each supernatant and kept at 4ºC for 24 hours, followed by centrifugation at 7000 ×g at 4ºC for 20 min. Each precipitates were dissolved in distilled water and precipitated again by repeating the previous steps and were dried until the mass reached a constant weight. Each experiments carried out in triplets and the ability of exopolisaccharide synthesis by the strains were evaluated in mg/L.

Two strains, *Lactobacillus plantarum* Mal and *Lactobacillus plantarum* K-2 found to produce 77 ± 1.4 and 81.4 ± 1.1 mg/l EPS, respectively, while EPS recovery in *Lactobacillus spp.* TM-5 was undetectable. The rest of the two strains found to produce EPS below 50 mg/l (table 1).

Table 1. EPS synthesis by several LAB strains.

| № | Strain | EPS Synthsis (mg/l) |
|---|---|---|
| 1. | *Lactobacillus plantarum* Mal | 77 ± 1.4 |
| 2. | *Lactobacillus plantarum* K-2 | 81.4 ± 1.1 |
| 3. | *Lactobacillus casei* P-1 | 45.7 ± 1.8 |
| 4. | *Lactobacillus spp.* TM-5 | - |
| 5. | *Lactobacillus agilis* Val | 15.7 ± 2..1 |

The results suggest each strains might have different EPS producing abilities or needs specific conditions for EPS synthesis. Moreover, it is known that EPS synthesis in LAB bacteria highly energy-dependant and LAB strains have an ability to generate energy in the limited amount and it might lead for limited EPS production [5].

Further research is planned on optimization EPS synthesis in LAB by changing the bio-chemical, technical parameters and increasing the carbon source percentage in the media.

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# EFFECTS OF ECG IMAGE PREPROCESSING ON THE CLASSIFICATION ACCURACY OF CONVOLUTIONAL NEURAL NETWORK


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The classification of ECG signals through convolutional neural network (CNN) networks has become one of the most effective directions today [1]. In most cases, the signal is processed as an image format in these networks. The format and formatting of these images can be further enhanced by improving the quality of the network by examining how images process (resizing, compression) can affect network accuracy. Therefore, the degree of impact of the image preprocessing on the clarity in this article is reviewed and analyzed.

In recent years, researches are beeing developt an effective way to use artificial intelligence applications or expert systems for each sector of economy. The world's most common expert systems have mainly developed in the field of medicine. Food and Drug Administration (FDA) has confirmed that the artificial intelligence applications which are specifically designed in the field of medical can only be a help to a doctor. Moreover, such applications are not yet at the enthusiasm level. Theories, knowledge and technologies have not been perfectly developed to create trustworthy applications. Therefore, creating of excellent expert systems in the field of medicine requires long and hard work.

In this paper ECG image preprocessing on the classification accuracy has been done on AlexNet and GoogleNet networks [2]. In the classification of ECG signals, AlexNet network is more tolerant than the image format of GoogleNet. GoogleNet presented better results when preprocessing with JPG formats, see table1.

Table 1

The accuracy changes of network classification in image preprocessing

| GoogleNet | GoogleNet | GoogleNet | AlexNet | AlexNet | AlexNet |
|---|---|---|---|---|---|
| From BMP format | BMP to JPG (not resizing) | BMP to JPG (not resizing) | From BMP format | BMP to JPG (not resizing) | BMP to JPG (not resizing) |
| 84,38 | 84,38 | 81,25 | 93,75 | 93,75 | 81,25 |

It is also aimed at examining the extent to which the ECG signal can be lost by resizing of the JPG and BMP format, and the impact on the accuracy of the network classification. It is known that the image quality is usually not corrupted when JPG file converted into BMP format. Otherwise, the image quality will be corrupted. For example, it's clear from results that when resizing the JPG image the efficiency of the network analysis has dropped. Resizing image was the same for JPG1 and JPG2 because BMP does not change image quality when converted to JPG format. Only, when formatting a BMP image before formatting JPG, it can be seen that the accuracy of network classification has dropped due to the loss of data needed for analysis [3].

Figure 1 illustrates the classification accuracy of the network when the image storage database is created in a variety of images of the ECG signal, generated by the normal viewing of the timeline. When the BMP format image was changed directly, the accuracy of network compatibility was 71.88%, the image was converted from JPG to BMP, and the result was changed to 75%. It is known that, when transferring a JPG image to BMP format, the normal image quality is corrupted and it affects the quality of the network too. But in this case we can see the opposite. It could be because, ECG signals, in contrast to the simplest picture, consisting of original signals and artifacts. Because of the frequency of ECG signals is often overlapped with the frequency spectrum of noisy, it is very difficult to remove all the noise.

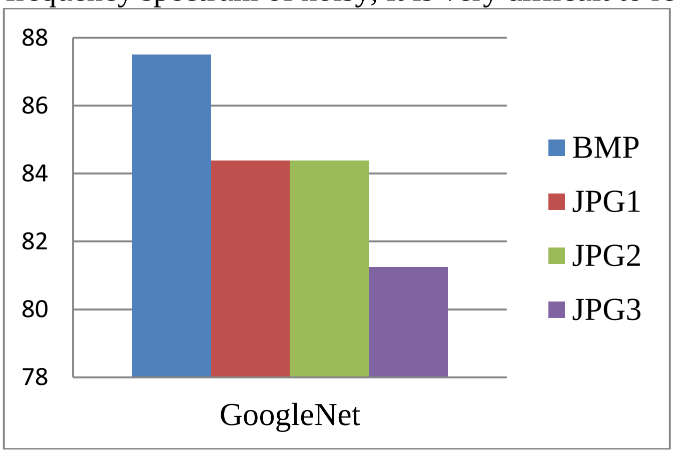

*Figure1. The accuracy of the network classification changes when preprocessing image*

Therefore, there are some smaller artifacts in the well-filtered ECG signals. After formatting from JPG into BMP view, it may be the considered noise levels have dropped. This situation can be likened to the Weight loss method used to improve network quality. In the same situation, the speed of the network has increased due to the taking off Weights. However, initially resizing JPG file size then transferring into BMP format it can be clearly seen that the result is diminished. Particularly, 12.5% reduction in AlexNet network was significantly detected. This can be summarized as shown in other figures that resizing of the image and also as a result of changing the picture format after changing the size may cause many data loss.

**Conclusion**

Usually, according to the CNN image recognition preprocessing, it can be taken into account that the images should not change pixels to the pixel many times. That is the cause of these events the data is compressed into filters and pools. This means that in CNN network any kind of format image is compressed many times then it is made up of a few points, in other word usually any image is compressed into the network. Therefore, preprocessing image formats is an important, that is it should be considered that it is durable or not durable for image compression.

Experience has shown that AlexNet network's format changing and accuracy of ECG classification is more stable than the GoogleNet network. In the classification of ECG signals, the network has been more successful than other formats when working with JPG formats. This difference was especially noticeable in GoogleNet network.

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## EVOLUTION METHODS FOR SOLVING OF INCORRECT PROBLEMS


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We will consider the transition to a finite approximation on the example of a Fredholm integral equation of the first kind [1-3]:

$$Az = \int_a^b K(x,s)z(s)ds = u(x), \quad c \le x \le d \,. \tag{1}$$

We assume that *K(x,s)* – a real function, continuous in the rectangle $\Pi = \{a \le s \le b,\ c \le x \le d\}$. We also assume for simplicity that the kernel *K* is nondegenerate. Let's say, instead of $\bar{u} = A\bar{z}$, we know it is the approximate value of $u_\delta$, that $\|u_\delta - \bar{u}\| \le \delta$, i.e. $U = L_2[c,d]$. From a priori considerations, we can say that the exact solution $\bar{z}(s)$, corresponding to $\bar{u}(x)$, there is a smooth function on *[a, b]*. For example, from a priori considerations $\bar{z}(s)$ that the exact solution $[a, b]$ and has nearly around corresponding is a smooth function on [*a, b*] with square. In this case it is natural to put $Z = W_2^1[a,b]$ [3].

We obtain an approximate solution of $z_\eta^{\alpha(\eta)} \in Z = W_2^1[a,b]$, which, when $\eta \to 0$ converges to $\bar{z}$ in the norm of $W_2^1[a,b]$, using standard scheme of the genetic algorithm.

To solve this problem is widely used following construction. We'll consider the functional

$$M^\alpha[z] = \|A_h z - u_\delta\|_U^2 + \alpha\|z\|_Z^2 .$$

Let $z_\eta^\alpha$ - extremal functional $M^\alpha[z]$, i.e. element, minimizing $M^\alpha[z]$ on Z. If parameter regularization $\alpha = \alpha(\eta)$ certain image is coordinated $\eta = \{\delta, h\}$, that element $z_\eta^{\alpha(\eta)}$ and will be in determined sense by decision of the problem (2).

In described production function $M^\alpha[z]$ for problem (2) is of the form of

$$M^\alpha[z] = \|A_h z - u_\delta\|_{L_2}^2 + \alpha\|z\|_{W_2^1}^2 =$$
$$= \int_c^d \Big[\int_a^b K_h(x,s)z(s)ds - u_\delta(x)\Big]^2 dx + \alpha \int_a^b \{z^2(s) + [z'(s)]^2\} ds. \tag{2}$$

At building certainly- differential to aproximations shall first of all choose nets $\{s_j\}_{j=1}^n$ and $\{x_i\}_{i=1}^m$ accordingly on length [*a*,*b*] and [*c*,*d*].

Hereon, using some quadrature by formula, can build certainly- differential analogue of the operator But integral equation (1). Certainly operator is a linear operator with matrix $A = \{a_{ij}\}$. Simplest variant to aproximations, which we shall use, is given formula.

$$a_{ij} = K_h(x_i, s_j),\ j = 2,...,n-1;\ \ a_{ij} = K_h(x_i, s_j),\ \ j = 1, n;\ \ i = 1, 2,..., m.$$

Now easy possible draw the функционал, aproximating (3):

$$\hat{M}^\alpha[z] = \sum_{i=1}^m \Big[\sum_{j=1}^n a_{ij} z_j h_s - u_i\Big]^2 h_x + \alpha\sum_{j=1}^m z_j^2 h_s + \alpha\sum_{j=1}^n (z_j - z_{j-1})^2 / h_s .$$

For finding of the extremum функционала $\hat{M}^\alpha[z]$, i.e. element $z_\eta^\alpha$ minimizing $\hat{M}^\alpha[z]$ on Z, is used genetic algorithm. The Genetic algorithm presents itself method эволюционного modeling applicable for decision of the problems with big dimensionality. Problem consists in minimization of the functions $\hat{M}^\alpha[z]$ on area of the possible decisions D in decision space Z.

Searching for of the decisions of the problem is in genetic algorithm to lead by means of populations by person. Each specimen is considered as pair генотипа g and images *z (z(g))*, which corresponds to certain point a space Z.

Genotip contains whole necessary information on corresponding to decision from Z. On base генотипа in nature are formed all external and internal signs of the person - фенотипы. In genetic algorithm of the collections all such sign corresponds to certain decision from Z. In proposed algorithm generation descendant begins with choice parental pair(vapour)s $< g_z(t), g_m(t) > \in \Pi^t \times \Pi^e$ at operator *Sel*. This operator has a probabilistic nature, showing a preference more suitable person.

To chosen genotypes with fixed by probability $p_c$ is used operator кроссинговера, replacing part gene one parent gene other. To got генотипу *g* or *h* is used operator to mutations. This operator by casual image changes the genes chosen генотипа on some casual symbols i.e. mutation introduces in the manner of random quantity *Mut(g)*, with distribution, hanging from g. Received hereon генотип changes генотип with worst importance $\hat{M}^{\alpha}[z]$ and process is repeated. For determination of the genetic algorithm necessary following components:

**1)** Alphabet, from which will is built genotype.

**2)** Image space point decisions in space genotypes

**3)** Function to fitness.

**4)** Operators GA

The Enormous influence upon velocity and quality of the decisions render the parameters of the genetic algorithm: power to populations and parameters operator. Usually they are selected to be empirical for given class of the problems.

Functioning(working) the algorithm presents itself consequent change population, consisting of fixed numbers by person, corresponding to point decision space. When making the next population more suitable persons leave more descendant moreover a part descendant will be an identical parent, but a part - suffers the changes to result of the mutations. At realization of the genetic algorithm for put(deliver)ed above problems much it is important to know importance of the target function on the best генотипе. Criterion of the stop of the process serves achievement required level of importance $\hat{M}^{\alpha}[z]$ or given amount iteration, but result of the algorithm serves the best specimen to populations.

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**NATURAL CHANGES AND TRAINING - DEVELOPMENT PROCEDURES**

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With a 4.5 billion-year-old genesis of the earth's sky-shaped history, it is known that the Koatservations with the first signs of life began their biography with the end of the Arche's era or the beginning of the Proterozoic era [1,2,3]. Due to his inability to the forces of nature, he tried to portray him in a different and logical manner. Explaining the structure of the universe in the size of the Earth, the Sun, the Moon, and the Stars has led to many mistaken ideas, ideas, views, assumptions and theories. Recalling the history of the only geocentric and the heliocentric doctrine is a clear example of this. Dialectics of life and death, health and diseases, the body and surrounding environment, as well as the relative scales of natural and artificial and synthetic world products, their prospects, biodiversity and bioenergetics, and many other controversies. Many global geological, tectonic, meteorological and other phenomena in the atmosphere, lithosphere and hydrosphere have led to a wide interpretation of the processes that take place on Earth. The complexity of the existing biosphere of the life unit has put new mysteries to the human race. Scientists have emerged, and some of the subjects that have been "discovered" for their research objects and objects, that is, clear and secular knowledge, have clarified many controversial issues. Socio-humanitarian disciplines open the laws of nature and society. Achievements in technology and technology, industry, and management have led to the creation of some of the world's private landscapes. However, some areas of knowledge - natural, technical, social and humanitarian sciences - can not form an integral part of nature, society and humanity, and their naturalistic phenomenon. The philosophical outlook will be clarified by the natural-scientific view of the universe. For this purpose, the most important achievements of nature science must be uniformity.

Before changing the universe, it is necessary to have a detailed understanding of it in detail. It is manifested in the creation of various images and essences of the universe. The first local view of the universe (mechanic, cosmological, physical, chemical, biological, private, social, anthropological, etc.) gave people only the incomplete impressions. That is why it is necessary to create a general scientific view of the universe. It’s shaping required not only interdisciplinary interdependent collaboration, but also the involvement of new and narrow, private and in-depth research that emerged in the core of the science that contributes to its formation [4].

As science is the highest form of knowledge, then objective truth rises to the level of eternal ideals. It is well known that the subjects have been divided into groups of large groups that study the living nature, inanimate nature, social spheres, according to their subject matter and research subjects. But it is a natural process to interact with different subjects that study material and spiritual world. At the same time, inevitably, sensitive specialization in each of these sciences is inevitably accompanied by the migration of research into narrow circles. The two objective trends in the development of science play an important role in the creation of a universal, scientific picture of the universe, as a result of differentiation and integration.

The most unique achievements of the civilization are achieved thanks to the integration of science. Understanding of the outside world and self, the knowledge of the new laws of human and society relations is happening. In such a dialectical process, the economic phenomenon of human intelligence is put on the leading position in production. When all efforts are focused on serving the health, dignity and superiority of a person, the essence of a large-scale vital phenomenon, such as production, becomes apparent. In order to live and reproduce, build and build, science and politics, production, farming and livestock should be established.

Due to the close relationship between the organism and the environment, the connection of celestial bodies, the transformation of life on the Earth by cosmic factors, the relevance of the changes in the life of the human society to the geological and astrophysical phenomena, the connection of the historical events of the Earth to solar activity, and their success in science it was. Such global scientific, creative, socially-educational and anthropo-non-oscillatory processes will continue uninterruptedly. The most important and common themes in "world-to-people" relations with their world outlook, methodological, axiological, gnoseological, ontological, pragmatic, humanistic, educational, communicative, critical, integrative, prognostic and sociological functions, and the philosophy and natural science will be accurate and complete, objectively and accurately interpreted in the light of truth.

When it comes to science-production, production, humanity, and the material and spiritual development of the world, there is a sense of purposefulness.

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# PURIFICATION AND ANTIMICROBIAL ACTIVITY OF *LACTOBACILLUS CASEI P-1* BACRIOCIN


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Bacteriocins are ribosomally synthesized antimicrobial peptides produced by bacteria, which kill or inhibit bacterial strains closely related to producer (1). The bacteriocins produced by lactic acid bacteria (LAB) have been the center of attention as they are generally regarded as safe (GRAS) and have potential application in the food industry. Further, bacteriocins have several attributes which make them suitable as food-biopreservative such as ability to inhibit pathogenic and spoilage bacteria, e.g., *L.monocytogenes, S.aureus, Bacillus cereus, Clostridium botulinum*, etc., susceptibility to digestive proteases, constancy in a wide range of temperature and pH, no alteration of the organoleptic properties of food, no toxicity to eukaryotic cells and simplicity to scale-up production (2).

*Lactobacillus casei* is gram-positive bacteria that is often included in the composition of staters for dairy, cheeses, and a number of other commercially fermented foods (3). The probiotic effects of *L.casei* is important both for the food and pharmaceutical industries.

Studies have reported that *L.casei* has potential probiotic traits such as growth rate and stability in commercial products, acidification ability, and favorable organoleptic properties.

Bacteriocin-producing bacteria exist widely in fermented vegetables. LAB are the dominant microorganisms in these fermented foods (4). *Lactobacillus casei P-1* has been isolated from the minced hot pepper Capsicum annuum.

*Lactobacillus casei P-1* bacteriocin was purified from culture media by three-step chromatography. *Lactobacillus casei P-1* was statically cultured in 10 mL MRS broth to $OD_{600}$ = 0.2, at which point 5 ml of the above culture was inoculated into 1000 mL MRS broth and then incubated for 48 h at 37°C without agitation. The fermentation culture was centrifuged at 7000 g for 30 min at 4°C to remove bacterial cells and then heated at 96°C for 15 min for proteases inactivation.

The SP-Sephadex C-25 cation exchange column (volume 50 ml) was equilibrated with 10 mM citrate buffer (pH 5.0). After equilibration, the crude bacteriocin was filtered through a 0.22 mm filter membrane and was loaded onto column. The loaded column washed with 50 mM, 100 мM, 200 мM NaCl and active fractions were eluted with 1 M NaCl. The flow rate was 3 mL/min. The active fraction cantained 130 mg of peptides and proteins while in L.casei P-1 culture media 4265 vg of proteins had been detected. At this stage, bacteriocin was purified approximately 32 fold and the yield of antimicrobial activity was 39% compared to culture supernatant.

The active fractions from cation exchange chromatography was filtered through 0.22 mm filter membrane, and loaded onto a Sep Pak C18 cartridge (volume 1.6 ml). At this stage, sodium chloride, soluble proteins and peptides, as well as some coloured substances, had been removed from the bacteriocin-containing fraction.

The active fraction from Sep Pak C18 cartridge was loaded onto Zorbax 300sb-C18 reverse phase column (5 mm, 4.6 x 250 mm, Agilent, USA) and chromatographed using high-performance liquid chromatography system by a linear gradient elution with water-acetonitrile (5–80%) containing 0.1% trifluoroacetic acid (TFA) in 40 min. The flow rate was 0.5 mL/min and the absorbance was monitored at 280 nm. The purified active fraction (retention time = 22.45 min) was evaluated for antibacterial activity. After HPLC re-chromatography, the bacteriocin purity was 95%. Highly purified bacteriocin P-1 from *Lactobaccillus casei* showed antimicrobial activity against indicator strains of *Listeria monocytogenes, Staphylococcus aureus and Pseudomonas aeruginosa* clinical isolates.

In the future, the molecular weight of the *Lactobacillus casei P-1* bacteriocin and it is amino acids sequence will be determined using the LC-MS. This, in turn, will allow to create the recombinant form of bacteriocin to increase the yield of antimicrobial peptide by recombinant bacterial strains.

Thus, the three-step chromatography protocol has been elaborated for purification of *L.casei P*-1 bacteriotcin.

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# APPLYING HAAR WAVELETS FOR CALCULATING THE OCTAVE ENERGY SPECTRUM OF SIGNAL

***H.N. Zaynidinov, I.Yusupov***

To estimate the convergence to the total energy of a signal of its spectral energy using the vector of squares of the coefficients, it is necessary to calculate the octave energy spectrum. Its advantage is the property of invariance with respect to signal shifts in time if they are stationary. This is a unique property that spectra have in the basis of complex exponential Fourier functions.

To obtain the integral value of the energy spectrum, it is necessary to calculate the totality of the sum of squares of orthonormal Haar wavelet coefficients taking into account the binary weights of the octaves:

$$E_{\varepsilon} = \left( \left(c_0^2 + c_1^2\right) + 2\left(c_2^2 + c_3^2\right) + 2^2 \sum_{k=2^2}^{2^3-1} c_k^2 + 2^3 \sum_{k=2^3}^{2^4-1} c_k^2 + \ldots + 2^q \sum_{k=2^{q-1}}^{2^q-1} c_k^2 \right) n. \qquad (1)$$

where $q$ is the largest order of iterations used in the implementation of the fast algorithm, and binary factors are necessary to take into account the weights of the octave orthogonal components in the total sum.

In world practice, a wide front is the development of methods for processing and interpreting digital material obtained as a result of measuring the intensities of fields of various physical natures. As an example of a specific function f(x) one variable, take the experimentally obtained function of one of the profiles of the magnetic induction field profile, measured in units of micro Tesla on one of the sections of the earth's surface. This area as a whole has a square shape measuring approximately 300 x 300 counts. The distances between the nodes of the two-dimensional lattice along the x and y axes are one kilometer each.

In order to perform examples of estimating the energy of high-frequency wavelet coefficients to calculate the signal sample length, we take the $n^0$2.1 function f(x) and make calculations for two cases:

- Sampling step h=2 signal and the number of samples in the sample is 64;
- Sampling step h=1, i.e the sample with double frequency and the number of samples of the same signal is 128.

The spectral energy calculations are performed in accordance with two options:

- At step h=2 (i.e 64 reference function and accordingly 64 wavelet coefficient) energy value is equal to $E_\varepsilon$=137.158;
- With double sampling rate (128 counts, step h=1, 128 coefficients) we get $E_\varepsilon$= 137.498. Thus, we can talk about the movement of the spectral energy of the sequence of wavelet coefficients to a certain value of the total energy.

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## ASSESSMENT OF FAMILIES AND LINES OF COTTON DERIVED FROM INTERSPECIFIC HYBRIDIZATION OF DIFFERENT BIOTYPES OF THE FUNGUS *FUSARIUM OXYSPORUM SCHLECHTEND. F. SP. VASINFECTUM*

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The area of our republic is damaged by the wilt, and the disease causes a decrease in cotton fertility and fiber quality. Particularly, in some years it is reported that the productivity of cotton is reduced by 20-25% or more by this disease. The emergence of new biotypes of wilt in recent years requires continuous research on this subject. Therefore, in addressing this problem, first of all, it is necessary to use different biological preparations to create new varieties that are tolerant to the disease and increase their resistance to the disease.

Fusarium wilt of cotton, caused by the fungus *Fusarium oxysporum Schlechtend. f. sp. vasinfectum* (Atk.) Snyd. & Hans, was first identified in 1892 in cotton growing in sandy acid soils in Alabama [1]. Although the disease was soon discovered in other major cotton-producing areas, it did not become global until the end of the next century. In addition to a worldwide distribution, Fusarium wilt occurs in all four of the domesticated cottons, *Gossypium arboretumL.*, *G. barbadense L.*, *G. herbaceum L.*, and *G. hirsutum L.*[2,3]. Disease losses in cotton are highly variable within a country or region. In severely infested fields planted with susceptible cultivars, yield losses can be high.

Symptoms of Fusarium wilt can appear at any stage of crop development. The cotyledons of affected seedlings wilt and die rapidly. Symptoms at this stage can be confused with symptoms of damping-off caused by *Pythium spp*., *Rhizoctonia solani Kuhn*, and *Fusarium spp*., but the brown vascular system of the hypocotyl distinguishes Fusarium wilt from seedling disease. The death of seedlings results in uneven stands, which further contribute to production problems throughout the season (Fig. 1). Symptoms in older plants include stunting, wilting, chlorosis and necrosis of leaves, dieback often beginning at the top of the plant, and plant death. (Figs. 2 and 3). Some affected plants may regrow from the base, but the new growth fails to produce bolls and often succumbs to the disease later in the season. Symptoms of Fusarium wilt are not always distinguishable from those of Verticillium wilt. The vascular discoloration is less pronounced and leaves often turn red in Verticillium wilt, but these differences are not reliable, and isolation of the pathogen is necessary to confirm the identity of the disease.

| | | | |
|---|---|---|---|
|  |  |  |  |
| Fig. 1 | Fig.2 | Fig. 3 | Fig. 4 |

*Fig. 1,2,3,4. Disease foci of dead and missing plants in a field infested by Fusarium oxysporumf. sp. vasinfectum.*

Many factors influence the development of Fusarium wilt in a field, including virulence of the population of *F. oxysporumf. sp. vasinfectum*, susceptibility of the cotton cultivar, climatic conditions, soil type, soil fertility, and interactions with nematodes and other soil borne microorganisms. All of these factors may impact the severity of the disease and subsequent yield losses. Visible symptoms may not be apparent when low inoculum densities of the fungus are present. In a greenhouse study, submerging root systems of susceptible cultivars in inoculum of *F. oxysporum f. sp. vasinfectum* did not result in visible aboveground symptoms until spore concentrations in excess of $10^5$ conidia per milliliter were used [4].

*F. oxysporum* includes a large number of pathogenic and saprophytic biotypes that share morphological features. Parasitic biotypes that cause Fusarium wilt of cotton were grouped into the forma specialis vasinfectum. Several races have been characterized within this forma specialis using both cotton and non cotton differential hosts. Recently, DNA-based techniques have been used to validate these races and to elucidate relationships among them. Armstrong and Armstrong's definition of resistance also was confusing, since they based susceptibility on the reaction in 50% or more of inoculated plants [2]. If more than 50% of plants of a cultivar developed external symptoms of wilt, it was considered susceptible in their assessment. A cultivar was considered resistant if less than 50% of inoculated plants developed external symptoms. Therefore, the demarcation between susceptibility and resistance of some cultivars was somewhat arbitrary, especially since a host's reaction was quite variable

In Uzbekistan alone, representatives of three groups were noted: races 1,3, 4. Today, race 1 is common in the Bukhara and much of Tashkent (for example, chirik , buka, and okkurgan), race 3 is common in , Bukhara, and Navoi, race 4 in Tashkent, Kashkadarya 2 and 4 in Samarkand, and the Uzbekistan biotypes are limited to that country. As researchers continue to use genetic tools to characterize more isolates, the distribution of various biotypes will no doubt be more widely distributed than previously assumed.

The obtained families and lines, with the participation of wild cotton species have comparatively studied the analysis of the sowing varieties, for resistance in production to different biotypes of *Fusarium oxysporum f. sp. vasinfectum wilt.* According to the results of the analysis, the C-4727 grade wasn't resistant to all biotypes and C-6524 grade was found to be not resistant to 4 out of 5 studied biotypes.

Reactions of cotton hosts for differentiating races of *Fusarium oxysporumf. sp. Vasinfectum*

| Biotypes | | | | | |
|---|---|---|---|---|---|
| Host ( .) | 1 | 2 | 3 | 4 | (Bukhara) biotypes |
| C-6524 | S | S | R | S | S |
| Namangan 77 | S | S | R | I | R |
| C-4727 | I | S | S | S | S |
| O-142-47 | S | R | S | S | S |
| O-107-16 | R | S | S | R | R |
| O-161-71 | S | S | S | R | S |
| O-117-25 | S | R | I | R | R |
| O-312-13 | R | R | R | R | R |
| O-132-41 | R | R | S | S | R |
| O-201-04 | R | I | S | R | R |
| CP-1303 | S | R | R | R | R |
| C-1306 | S | S | I | S | S |
| O-87-01 | R | R | S | R | R |
| T-1379 | S | R | R | R | R |

In some studies, certain cultivars were substituted for the ones shown. S = susceptible (wilt symptoms), R = resistant (no wilt symptoms), and I = intermediate (some plants susceptible).

Among the varieties resistant to disease is shown in Namangan-77 grade, which is found to be not resistant to 2 biotype (3.4). Among the families and lines of wild-growing species, the O-312/13 family was found to be resistant to all biotypes, while T-1379, O-87-01, and CP-1303 were found to be tolerant, 4 of 5 studied biotypes. The relative negative outcome was observed in C-13-06, with all the biotypes being susceptible. The family O-161-71 had 4 biotypes, and the family O-142-47 had 2 biotype is resistant than all other biotypes. In general, most of the studied families and lines have been found to be resistant to various biotypes of *Fusarium oxysporum f. sp. vasinfectum.* The main reason for this is that family and rocks can be characterized by the presence of wildlife species that is resistant to the disease, that is, the genotype effect. In conclusion, it is possible to create genetically enriched, naturally resistant families and lines with the involvement of wildlife species.

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## FORMATION OF STUDENTS' ENVIRONMENTAL COMPETENCIES IN PHYSICS LESSONS

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Due to the development of technology and technology, the educational process will be improved, and the application of the achievements of scientific and technological progress gives a new force in the development of the education system. We have analyzed the development of these periods and the comparison, which helped us to draw the following conclusions of the pedagogical research.

Pedagogical research conducted by us revealed that over the last 20-25 years, the structure of curricula, programs and teaching standards has changed significantly in comparison with the development of pedagogy, engineering and technology. During this period, advanced science, technology and technology. For the subsequent period, new pedagogical teaching aids are used in physics classes.

This article considers the issue of the impact of technical development on the formation of students, where it is necessary to take into account the development of modern physics and information technologies that meet the present requirements of the time. [1]

Our experiments revealed the dependence of sounds and noise on pupils. When studying physics in the 6 th grade, the students are confronted with such topics as the "Sound Sounds and Noises" section of the sound phenomena section of the subject of physics.

Sound is a physical process that is considered to be the main fundamental environmental factor adversely affecting the environment. That is why the concept of noise should be taught to students from the point of view of physics. This should be done with the help of knowledge gained by students on other subjects and their improvement.

Also, the concept of noise is considered to be one of the components of environmental pollution associated with physics; today, every second person feels its effects on himself [2]. The noise reaches the land where the human foot has not yet touched, with a clear sky and clear water. Experts associate a sharp increase in diseases of the cardiovascular system, gastrointestinal and hypertensive among the population. In addition, an increase in the noise level by 1% reduces the production efficiency by the same amount, apparently, noise is one of the dangerous environmental factors for the human body.

Noise acts so much on the human body, but also on animals, insects and plants, for example, trees under the influence of noise age earlier than trees in the forest, under the influence of the noise of a jet plane, the larvae of bees die, and they themselves lose their orientation, etc.

In large countries, the level of normal sound pressure is 65-80 dB, while it should be 50-55 dB outdoors, at 130 dB a person will feel severe ear pain, and the sound of 154 dB cannot be sustained at all. In the latter case, the person has a severe headache, he becomes stuffy, the visual organs refuse.

According to the researchers, the noise intensity in cities increases 10 times every 25-30 years. This means that the noise level has increased by 10 dB.

Familiarize students with these materials can be when studying physical laws. But since the study of these materials requires a lot of time to explain them, both during lessons and extracurricular activities [3]. Pupils should realize that in crowded places, in medical institutions one cannot use technical means that make a loud sound.

For a complete (extensive) concept of noise, laboratory work is organized on the topic "Determining the speed of sound in air depending on temperature"

In this experiment, the speed of propagation of a sound pulse in air is determined, and the group and phase velocities coincide with the speed of sound. The sound pulse is generated by dust-like tension "vibration" of the speaker membrane. These movements of the dynamics lead to fluctuations in air pressure. The sound pulse is recorded at a certain distance from the speaker.

To determine the speed of sound υ, we measure the time span between the speaker generating sound and microphone recording sound. Since the exact initial phase of the sound pulse cannot be accurately determined, the measurement is taken from two places from the microphone at distances $s_1$ and $s_2$.

The speed of sound is defined as the ratio of the difference of distances $\Delta s = s_1 - s_2$ to the difference in time of flight $\Delta t = t_1 - t_2$, as $\upsilon = \Delta s / \Delta t$.

The device for sound and speed allows you to heat the air with a heater; At the same time, this device blocks surrounding effects, such as temperature differences and air convection, which can affect the measurement.

In this system, pressure P remains constant (in fact, it is equal to atmospheric pressure). With increasing temperature T, the air density ρ decreases, and the speed of sound c increases.

The order of the experiment

Measurement at room temperature

Run the installation (Load settings)

• Save multiple individual measurements with

• Slide the universal microphone all the way into the plastic tube and register the change in distance Δs on the metal rail with the scale.

• Save multiple individual measurements with

• Determine the speed of sound using $c = As/At$ (determine the average time of flight in the graph using Draw Mean).

a) Measurement depending on temperature

Run the installation

• Slide the universal microphone.

• Again at room temperature, determine the $At_{A1}$ transit time, using the already determined sound velocity, calculate the distance $s = c_A t_{A1}$ between the microphone and the speaker and record these values in the table (click on the first row in the s-column).

• Connect the heating thread to a voltage source (12V/approx. 3.5 A) through the slots in the apparatus cover.

Save the current flight time from (for example, every 5°C).

Evaluation

Once you determine the speed of sound at room temperature in

and thus the distance s between the microphone and the speaker in the software calculates the correct sound speed simultaneously with each time span of $A_t A_1$

Sound speeds are recorded in the Temperature display as a function of temperature, during measurement. By fitting on a straight line you can easily confirm the literary value of *υ=(331.3+0.6-9/°C) m/s.*

In this laboratory work, students study the Definition of the speed of sound in air depending on temperature. At the same time, it recognizes the dependences of the speed of sound on temperature, which results in improvements in the competence of students.

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## ACTION OF SULFODERIVATIVES-DB18C6 ON THE CONDITION OF MULTILAMELLAR LAYERS


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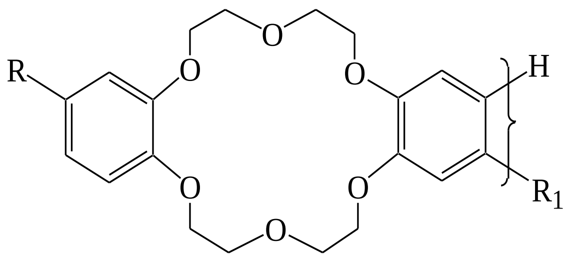
The study of the interaction of crown-ethers, which different in structure and mechanism of membrane action, with multilamellar dispersions makes it possible to determine the difference in their effect on the rearrangement of the initial structure of lipid bilayers.

Sulfoderivatives-DB18K6 were synthesized in the Laboratory of macrocyclic compounds of the Faculty of Chemistry of NUU by prof. Tashmukhamedova A.K. et al [1].

Multilamellar dispersions from DMPC (dimyristoylphosphatidylcholine) were prepared in 10 mmol Tris-HCl buffer solution (pH = 7.5) with a lipid concentration of $3*10^{-4}$M.

Investigated compounds (Fig.1): Water-soluble sulfo derivatives-DB18C6: 4'-tretbutyl-4 "(5")-DB18C6 sulfonic acid, 4'-acetyl-4"(5")-DB18C6 sulfonic acid, 4-DB18K6-monosulfonic acid and 4',4"(5")-DB18K6 disulfonic acid, show $K^+$-channelformer properties on bilayer lipid membranes [2,3,4].




$K^+$- channelformers: sulfoderivatives - DB18K6 [4]

$R = –C(CH_3)_3$;  $R_1 = –SO_3H$ 4’-tret-butyl-4’’(5’’)-DB18C6-sulfonic acid (1)
$R = –COCH_3$;  $R_1 = –SO_3H$ 4’-acetyl-4’’(5’’)-DB18C6-sulfonic acid
(2)
$R = –H$;  $R_1 = –SO_3H$ 4’-DB18C6-sulfonic acid (3)
$R = –SO_3H$;  $R_1 = –SO_3H$ 4’,4’’(5’’)-DB18C6-disulfonic acid (4)

*Fig.1. Structural formulas and names of the studied crown-ethers.*

It was previously established that water-soluble sulfoderivatives - 4'-tretbutyl-4"(5") - DB18C6-, 4'acetyl-4"(5")-DB18C6-, 4'-DB18C6- sulphonic acid and 4',4”(5”)-DB18C6 disulfonic acid is capable of inducing single ionic conduction channels for monovalent cations in lipid bilayers [2,4]. The channelformer properties of these compounds depended on the degree of their lipophilicity and were arranged in the following row: 4'-tretbutyl-4"(5") - DB18C6 sulfonic acid> 4'-acetyl-4"(5") - DB18C6-sulfonic acid> 4'-DB18C6-sulfonic acid> 4',4”(5”)-DB18K6 disulfonic acid [2,3].

The studied sulfoderivatives-DB18K6 lead to a change in the initial organization of lipid molecules, which, first of all, reduces the magnitude of the pretransition peak. This is observed when more lipophilic sulfoderivatives — 4’-tretbutyl-4"(5")-and 4'-acetyl-4"(5")-DB18C6-sulfonic acid are added to the lipid dispersion from DMPC. Practically for these crown-ethers, even at the $C_c/S_{lip}$=0.02, the pre-transition peak disappears, whereas in the case of 4'-DB18C6-sulfonic acid and 4',4"(5")-DB18C6 disulfonic acid compounds, the pre-transition peak is observed at all concentrations used . Consequently, crown-ethers 4'-tretbutyl-4"(5")-DB18C6-sulfonic acid and 4'-acetyl-4"(5")-DB18K6 sulfonic acid lead to a decrease in the total enthalpy of the phase transition process by 17.7%, whereas for 4'-DB18K6-sulfonic acid and 4',4”(5”) - DB18C6-disulfonic acid, this decrease is 6.3% and 2.6%, respectively.

It was shown that 4'-DB18K6-sulfonic acid and 4’,4”(5”)-DB18K6 disulfonic acid have a weak influence on the thermodynamic parameters of the phase transition process of the DMPC dispersion. In the case of the interaction of $K^+$ channalformers: 4'-tret-butyl-4 "(5") -, 4'-acetyl-4 "(5")-, 4'-DB18C6-sulfonic acid and 4',4"(5")-DB18K6 disulfonic acid with multilamellar dispersions from DMPC was observed as the expansion of the main melting peak, and the temperature shift of the main phase transition towards low temperatures.

It was established that the interaction of $K^+$ channelformers with multilamellar dispersions from DMPC exhibits both an expansion of the main melting peak and a shift in the temperature of the main phase transition towards lower temperatures.

Based on the data obtained, it can be stated that $K^+$ channelformers: 4'-tretbutyl-4"(5") - DB18C6-sulfonic acid, 4'-acetyl-4"(5")-DB18C6-sulfonic acid, 4'-DB18K6 sulfonic acid and 4',4"(5")-DB18C6-disulfonic acid reduce the overall enthalpy of the phase transition of the process of melting lipids, reducing the cooperativity of the process.

With an increase in the relative concentration of crown-ethers to lipid, the value of the half-width of the phase transition increases, while the temperature of the main melting peak of lipids $T_{mel}$ remains unchanged. Apparently, $K^+$ channelformers violate the strict packing of hydrocarbon chains of phospholipids, without causing any noticeable surface perturbations ($T_{mel}$=const).

For sulfoderivatives-DB18K6, which induce $K^+$ channel conduction of the type of “molecular associates”, an increase in the cooperativity of the phase transition of lipids is not observed, but rather a decrease in the cooperativity of the phase transition of lipids, although, like in the case of the lipid channel, we are dealing with inducing channel structures. This can serve as another strong argument in favor of the difference in the mechanisms of these two types of channeling by crown-ethers on bilayers.

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# ACTION OF LOW-FREQUENCY ELECTROMAGNETIC FIELDS ON THE CONTENT OF INTRACELLULAR CALCIUM IN WHEAT PROTOPLASTS

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To explain the biological effect of super-weak low-frequency electromagnetic fields (LF EMF) on living organisms, many hypotheses have been proposed, the most popular of which are the so-called "resonance" ones, in which the calcium ion plays the role of the main target.

According to these hypotheses, for the implementation of many biological processes, it is necessary that the ions participating in them, in addition to the thermal "Brownian" movement, also make oscillations (trembled). These oscillations arise because all ions move in a constant geomagnetic field (GMF), and any charged particle moving in a magnetic field tends to move in a spiral with a frequency that is called cyclotron. With the magnetic induction of the constant component of the GMF-50 μCT, the cyclotron frequencies of most biologically important ions lie in the range of 1–50 Hz. Although induced GMF changes in the kinetic energy of ions are much less than the energy of thermal fluctuations, it is assumed that GMFs significantly swing ions at times when they are in nonequilibrium conditions, for example, when substrate ions are in the active center of the enzyme or when an ion passes through the portal membrane channel mechanism. Thus, the oscillating ion quickly passes through the channel of the biological membrane, and the oscillating substrate molecule quickly finds the desired key-lock position in the active center of the enzyme (Uzdensky, 2000).

The most important regulatory ion in the cell are calcium ions, as they are intracellular messengers. Outside the cell in the intercellular space, the concentration of calcium ions is $\approx 10^{-3}$ M, and inside it is $10^{-7}$M. To maintain such a low concentration of calcium ions inside the cell, plasma pumps, membranes of the sarcoplasmic reticulum and mitochondria contain active pumps or ion exchangers, pumping calcium out or pumping into the reticulum and mitochondria. Even a small increase in the concentration of intracellular calcium activates many calcium-dependent enzymes and processes.

The modern method of registering small changes in the concentration of intracellular calcium in the range of concentrations of $10^{-7}$–$10^{-6}$M in living cells is the fluorescent probe method. It is based on the property of special molecules Fura-2, Indo-1, chlortetracycline (CTC) to fluoresce when attaching calcium ions to them (Levitsky, 1990).

If the resonance hypotheses about the effect of LF EMFs on biological ions are correct, then the concentration of calcium ions in the cytoplasm of plant cells will vary depending on the frequency of the external EMF.

The purpose of this work was to test these resonance hypotheses on a plant cell model, protoplasts from wheat leaves.

**Materials and methods.** The protoplasts were isolated from the leaves of 6-8 day old wheat seedlings using a 2.5% cellulase enzyme, as described in the Large Workshop on Plant Physiology (1978). The concentration of protoplasts was determined by counting cells in the Goryaev chamber. Cell viability was assessed by cell color of 0.04% trypan blue, it was at least 95% in all experiments. The cell density in the cell was 106 cells / ml.

The protoplasts were incubated in medium containing 150 mM Sucrose, 150 mM $Na_2SO_4$, 1 mM $CaCl_2$, 10 mM Tris-MES-buffer pH = 7.1 and 20 µM Indo-1 for 1 hour.

The final cell concentration in the measuring cell of the fluorimeter with fluorescence was $5{\cdot}10^5$ cells / ml for protoplasts. The fluorescence intensity of Indo-1 at 405 nm was recorded on an SFR-1 spectrofluorometer (Pushchino, Russia) at 25°C. Excitation wavelength - 334 nm.

Loaded Indo-1 protoplasts were placed in the measuring cell of the fluorimeter in a medium with 1 mM $Ca^{2+}$ and were treated with pulsed EMF in the range from 1 to 100 Hz.

For the treatment of plant material EMF used a homemade electromagnetic pulse generator, creating a damped burst of pulses with a repetition rate of 1-100 Hz and a magnetic induction of 10 µT.

**Results and its discussion.** It was shown that, starting from a frequency of 2 Hz, abrupt transient changes in fluorescence occur, reflecting abrupt changes in the concentration of $Ca^{2+}$ in the cytoplasm of cells called calcium oscillations.

With further increase in frequency, these oscillations merge, forming different frequency-dependent fluorescence levels, which decrease when the generator is turned off. The figure shows the dependence of the fluorescence intensity on the frequency in the range of 1–100 Hz, from which it can be seen that the fluorescence intensity is maximum at frequencies of 1–50 Hz with peaks of 16, 32 and 64 Hz.

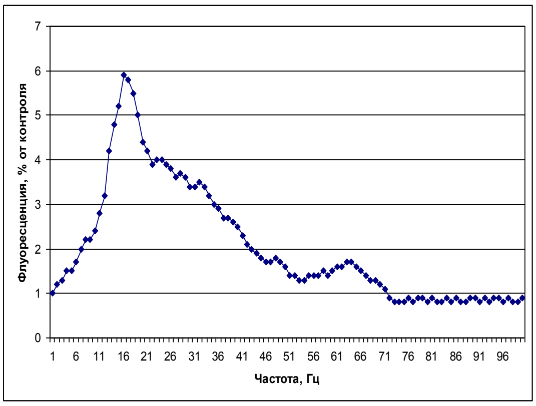


*Fig. The dependence of the fluorescence of Indo-1, loaded into wheat protoplasts, on the frequency of EMF.*

The magnetic induction of the Earth's constant magnetic field in the body of the fluorometer, in which we measured the probe fluorescence, was between 15 and 25 μT (measured with the Samsung Galaxy S smartphone). The cyclotron frequencies for calcium calculated for this magnetic induction lie in the range of 11–19 Hz. The frequency of 16 Hz obtained by us fits well into this range and is even more complete proof of the cyclotron hypotheses.

## CORRECTION OF QUANTITATIVE CHANGES OF PHOSPHATIDYLCHOLINE, PHOSPHATIDYLETHANOLAMINE AND THEIR LISOFORMS IN LIPID PEROXIDATION IN MITOCHONDRIA BY HAPLOGENINE-7-GLUCOSIDE

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The importance of lipid peroxidation is that they form one of the physiological modifications of the phospholipids in bilayer lipid biomembranes, participate in regeneration processes of phospholipids and their structure [1].

The change phospholipid's concentration in the mitochondrial membranes results the structural and functional changes in the membrane. As a result, there are changes membrane's ion transport, the activity of the receptors, oxidative phosphorilation, and the activity of the various enzymes. Therefore, we focused to investigate the structural changes in phospholipids when the mitochondria were incubated with haplogenine-7-glucoside under lipid peroxidation condition.

Experiments were carried out on male white rats with 160-180 g. weight. Animals were left hungry for one day before experimenting. Mitochondria have been isolated from liver using a differential centrifugation [2]. Extraction of mitochondrial phospholipids and their composition are made by Blay and Dayer [3]. The phospholipid composition of the mitochondria was analyzed by two-dimensional micro-duplicate chromatography [4].

Nonenzymatic Fe2+/ascorbate-induced lipid peroxidation was carried out of 0,25 M sucrose, 10-5 M FeSO4 and 2×10-4 M in an aqueous medium with a molar 1 mg of 6-8 mg of mitochondrial suspension. In the composition of the alcohol solution the hydrophobic dien conjugates [5] and the amount of phosphorus were determined. The protein content of the mitochondria was determined by the method of Lowry [6]. The results obtained were calculated using Student-Fisher's statistical method, average arithmetic mean (M), error (m), reliability index (T and P). P was used as a reliable indicator of difference in size less than 0.05.

The results obtained by correction of phosphatidylcholine, phosphatidylethanolamine and phosphatidylethanolamine in lipid peroxidation process of mitochondria with haplogenine-7-glucoside are given in тable 1.

Table 1

Correction of quantitative changes of phosphatidylcholine, phosphatidylethanolamine and their lisoforms in lipid peroxidation in mitochondria by haplogenine-7-glucoside

| Data | Incubation period, min. | Autooxidation | $FeSO_4$+ ascorbate | $FeSO_4$+ascorbate+ 60 micg/mg peptide |
|---|---|---|---|---|

| | | | | haplogenine-7-glycoside |
|---|---|---|---|---|
| phosphatidyl-choline | control | 39,4±1,5 | 39,4±1,5 | 39,4±1,5 |
| | 30 | 36,4±3,1 | 44,4±1,8 | 41,7±1,9 |
| | 60 | 35,7±2,4 | 52,4±1,7 | 44,3±2,2* |
| | 90 | 34,6±2,5* | 54,5±1,8 | 47,0±2,0*** |
| lisophosphatidyl-choline | control | 1,99±0,19 | 1,99±0,19 | 1,99±0,19 |
| | 30 | 2,65±0,11* | 3,46±0,21*** | 2,68±0,41*** |
| | 60 | 2,80±0,13* | 4,74±0,29*** | 3,29±0,59*** |
| | 90 | 2,95±0,11* | 5,53±0,36*** | 3,72±0,88*** |
| phosphatidyl-ethanolamine | control | 32,1±2,5 | 32,1±2,5 | 32,1±2,5 |
| | 30 | 31,7±1,7 | 18,9±1,9*** | 27,4±2,3* |
| | 60 | 27,6±1,6* | 15,6±2,4*** | 23,4±2,3*** |
| | 90 | 26,4±1,4** | 13,6±2,8*** | 21,0±4,1*** |
| lisophosphatidyl-ethanolamine | Назорат | 6,16±0,56 | 6,16±0,56 | 6,16±0,56 |
| | 30 | 5,69±0,25 | 5,13±0,45* | 6,00±0,78 |
| | 60 | 5,57±0,21 | 4,40±0,28*** | 5,33±0,41* |
| | 90 | 5,47±0,34 | 3,86±0,15*** | 4,92±0,54*** |

*Note: (*P<0,05; **P<0,01; ***P<0,001)*

Mitochondrial LPO with haplogenine-7-glucoside was incubated for 60 minutes and 90 minutes. Phosphatidylcholine indicator depends on time ralativly to FeSO4+ascorbate and decreased to 6.1; 15.5% and 13.8% respectively. Lisophosphatidylcholine, with time delayed control, increased the LPO activity, while the peroxidation of haplogenin-7-glucoside resulted in compared to FeSO4 + ascorbate during 30, 60 and 90 was decreased to 22.6, 30.6 and 32.8% respectively. The indicators of phosphatidylethanolamine and lisophosphatidylethanolamines with relative haplogenin-7-glucosidase have been shown to have a sharp decrease in comparison to FeSO4 + ascorbate.

The amount of phosphatidylethanolamine after incubation for 30; 60 and 90 minutes was 14.7; 27.1% and 34.6% respectively compared to control in mitochondria and haplogenin-7-glucoside condition. The amount of lisophosphatidylethanolamine during the incubation period in 30 minutes did not change, but decreased in 13.5 and 20.1% in 60 and 90 minutes (Table 1). Thus haplogenin-7-glucoside reduces the effect of ROS on lisophosphatidylethanolamine and in particular phosphatidylethanolamines.

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## ROLE OF THE BIO CONTROL AGENTS BY THE LIMITING OF THE SUNN PEST (*EURYGASTER INTEGRICEPS* PUT.) POPULATION IN TASHKENT REGION.

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*Eurygaster integriceps Put* affects to wheat and barley by eating their stems, especially suffering autumn wheat in Central Asia [6]. The irrigation and mountain areas *Eurygaster* eggs may be contaminated egg parasites to 70% [3], 80% [1] and even 90% [4].

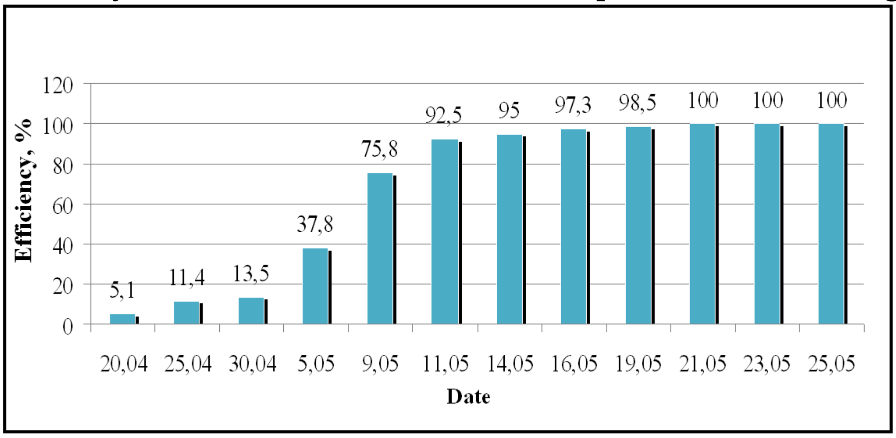
In the study of natural Sunn Pest entomophages-fly Phasii was found serious damage to the development of the Sunn Pest imago population [2, 5]. The Sunn Pest parasites of its eggs and adults were studied in several places of the Tashkent region.

To limit the amount of high populations of plays important role as biological agents eggs parasites.

Egg parasites are considered an important biotic constraint during the formation of the population of a new generation of Sunn Pest. The emergence of the larvae from the eggs of Sunn Pest in nature is observed after 10 days of laying. First-stage larvae were observed on April 25 and lasted until May 18. First instar larvae were round and dark color (Figure. 1).

**Figure 1**

**The dynamics of infection with intestinal parasites Sunn Pest eggs.**




Eggs Sunn Pest in the first decade of May, very rapid pace infected egg parasites and since this period, the formation of pre-larvae of Sunn Pest decreases sharply (Figure. 3). Sexual reproduction of Sunn Pest in the period 19.05-30.05 rises sharply and the highest rate was observed 27.05. During the very high rate of sexual reproduction of the *Eurygaster* observed that sexual reproduction of 70 *Eurygaster* reached 450 eggs in a day. Despite the fact that sexual reproduction Sunn Pest during this period were the highest rate, laid eggs were infected with eggparasites is 100%, and destroyed.

Departure egg parasites from infected eggs was observed after an average of 12-14 days, and hatching eggs of parasites was 98-100%. During the season Sunn Pest laid eggs were parasitized an average over 30%.

Indicators of contamination of eggs *Eurygaster* intestinal parasites in the first ten days of May were high and in the second decade of the indicators have reached 100%. Infected eggs are different from uninfected Sunn Pest that infected eggs have black.

As parasites Sunn Pest imago play a special role as a fly Phasii. During research in wheat fields were discovered three species of parasites Sunn Pest imago flies Phasii: *Helomiya lateralis Meig.*, *Phasia subcoleoptrata* and *Phasia crassipennis* were studied their biological effectiveness. Sunn Pest flies appear was observed into place migration from the winter to wheat fields of adult individuals in the Sunn Pest body. The larvae appear is in the body cavity and changing to the pupa in the soil. Adult phase of Sunn Pest were observed in April and May. Females flyblow into middle place of head and chest parts of the adult Sunn Pest. Both rounds of the phase out of flying insects were in late April and early May. Biological effectiveness was studied of *Phasia crassipennis* in natural conditions (Picture 1) and it was found to be higher than *Helomaga lateralis Meigs*. and *Phasia subcoleoptrata*. In general, Sunn Pest imago parasite in Tashkent region was noted that there is no the high biological activity.

**a b c**

**Picture 1. Infected Sunn Pest with *Phasia crassipennis* (a), pupa (b) and imago (c), (orig.)**

This Sunn Pest's pupa period is 16-18 days. They were living in an average of 9 days during the flying time when they made care by the sugar solution. The following in Tashkent region conditions harmful Sunn Pest was infecting with Phasii flies of biological efficiency.

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# SCIENTIFIC BASES OF THE SELECTION OF TYPES PLANTS FOR PHYTORECULTIVATION OF TECHNOLOGICAL PLANTS OF THE CUTTING OF ANGRENA WAY SYSTEM ANALYSIS OF ADAPTATION FEATURES OF PLANTS

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The protection of vegetation cover is currently one of the urgent problems of our time. In connection with the ever increasing of the negative influence of the anthropogenic factors on natural ecosystems, great attention must be paid to the protection and restoration of disturbed natural ecosystems for sustainable development. In the Uzbekistan, despite the fact that the area of disturbed lands is increasing, research work on the restoration of the vegetation of disturbed lands is being done insufficiently. Currently, the area of disturbed lands in the country is increasing.

The purpose of our research is the scientific substantiation of the selection of species for man-made disturbed lands.

This is necessary to solve the problems of ecological optimization of disturbed lands by creating rain-fed pasture-hay crops for phytocenoses, as well as improving soil fertility for further agricultural use.

The selection of species for the phyto-recultivation of technogenically disturbed lands was made on the basis of applying the method of system analysis of the adaptation features of plants in the arid zone of Uzbekistan [1,2].

Euxerophytes;

- *Kochia prostrata subsp. grisea.*
- *Kochia prostrata subsp.verescens.*

Teriremoxerophytes;

- *Artemisia ferganensis H.Krasch.*
- *A. serotina Bunge.*

Xeromesophytes;

- *Melilotus officenalis.*
- *Alcea nudiflora (Lindl.) Boiss.*
- *O. chrassanica Bunge.*
- *Medicago tianshanica Vass.*

Mesophytes;

- *Hordeum bulbosum L.*

The studied species are classified into ecological types of xerophytes and mesophytes. According to the climatic conditions, the “Angrenskiy” section refers to the submountain semi-desert zone. The average perennial amount of precipitation is 500 mm.

The soil mixture at the experimental site is represented by kalinin, variegated neogene clays, limestones, sandstones with a 40-50 cm overlapped top layer of brown coal particles displaced with soil.

On the disturbed lands of the rock heaps of the Angrensky open-pit mine, the site of the Appartak dump was chosen for the experiments.

Technical training of the experimental site was to carry out the planning and cleaning of stones. An area of 1 hectare was blocked by loess with a thickness of 20–25 cm. The sowing was done in rows, the length of the rows was 20 m, and the aisle was 60 cm. Plants were placed on the plot along families in separate maps.

As a result of the studies carried out on the phyto-recultivation of technogenic lands in conditions of dry rash, representatives of the ecological groups of euxerophytes, teriorimexerophytes, xeromezophytes and mesophytes were promising.

The best growth and development in the conditions of the rock heaps were characterized by: *Kochia prostrata subsp. grisea., Artemisia ferganensis H.Krasch., A. serotina Bunge., Alcea nudiflora (Lindl.) Boiss., Medicago tianshanica Vass., O. chrassanica Bunge., Hordeum bulbosum L.*

Adaptation of the water regime of plants to the transfer of the xerothermic period in the studied species is expressed in an increase in osmotic pressure and water holding capacity (Table 1).

**Table 1**

**Indicators of water regime in the conditions of the rock heaps of the Angrensky open-pit mine**

| № | Name of the plants | Water content, % | Water deficiency, % | Water holding capacity, % | Osmotic pressure, atm |
|---|---|---|---|---|---|
| 1 | *Kochia prostrata subsp. grisea.* | 81,0-62,0 | 11-21 | 56,0-80,6 | 6-29 |
| 2 | *Kochia prostrata subsp.verescens.* | 81,0-66,2 | 11-28 | 63,0-82,0 | 7-34 |
| 3 | *misia ferganensis H.Krasch.* | 74,2-62,3 | 16-36 | 74,5-92,0 | 6-17 |
| 4 | *A. serotina Bunge.* | 75,2-59,3 | 12-38 | 81,0-94,0 | 9-20 |
| 5 | *Melilotus officenalis.* | 81,4-76,5 | 12-15 | 69-30 | 12-24 |
| 6 | *Alcea nudiflora (Lindl.) Boiss.* | 76,6-63,6 | 18-29 | 54-29 | 9-22 |
| 7 | *O. chrassanica Bunge.* | 74,3-63,0 | 11-20 | 74,0-45,0 | 15-46 |
| 8 | *Medicago tianshanica Vass.* | 77,8-74,0 | 19-21 | 65-76 | 16 |
| 9 | *Hordeum bulbosum L.* | 74,7-69,4 | 11-39 | 26,5-30,5 | 6-11 |

Biological adaptation of barley is expressed in terms of their vital activity for the mesoterm period. The onion barley also has structural adaptation - the presence of pseudobucus, and in lucerne, some aspects of functional adaptation (an increase in water-holding capacity and osmotic indices).

In plants with a long vegetation period, adaptation to the xerothermic season is expressed in a change in the generation of mesophilic leaves to more xeromorphic ones, an increase in water-holding capacity.

Bulb barley has a demutational ability - by spreading self-sowing they contribute to the restoration of the former vegetation of disturbed lands.

The study of root systems showed that plant roots penetrate into the breed from the second year of vegetation. The most superficial root system in bulbous barley (up to 50 cm) is the number of pseudo-bulbs up to 6-8. The most deeply penetrating root system is in the stem of a rose (155 cm). Powerful root systems with lateral branches are noted in the subspecies of imen and species of wormwood. In species of sainfoin, there are many nitrogen-fixing nodules in the roots, the remains of which enrich the soil with nitrogen.

The yield of forage mass is gray up to 63 c / ha, seeing polyny up to 58 c / ha, sainfoin up to 46 c / ha, rose stem 161 c / ha, bulbous barley 12 c / ha.

For the phytorekultivation of the rock heaps of the Angrensky open-pit mine, *Kochia prostrata subsp. Grisea, Kochia prostrata subsp.verescens, A. serotina Bunge., Artemisia ferganensis H.Krasch., Alcea nudiflora (Lindl.) Boiss., Medicago tianshanica Vass., O. chrassanica Bunge., Medicago tianshanica Vass., Hordeum bulbosum L.* were promising.

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## COOPERATION TECHNOLOGY OF HIGHER EDUCATION INFLUENCE OF FRAMEWORK COMPETITIVE COOPERATION

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This article highlights the technology of family cooperation in upgrading professional skills of future professionals in higher education institutions, in which the role of the family in the education of young people today, the technology of establishing collaborative links in the formation of their professional competence, and the relationships with the family's higher education institution in improving professional competence in foreign higher education institutions analysis of the results of research and the factors that influence the quality of education imposed.

# DATA SCIENCE

## Plenary Lectures

### STRUCTURE CHOICE FOR RELATIONS BETWEEN OBJECTS IN METRIC CLASSIFICATION ALGORITHMS

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We analyze the cluster structure of learning samples, decomposing class objects into disjoint groups. Decomposition results are used for the computation of the compactness measure for the sample and its minimal coverage by standard objects. We show that the number of standard objects depends on the metric choice, the distance to noise objects, the scales of the feature measurements, and nonlinear transformations of the feature space. We experimentally prove that the set of standards of the minimal coverage and noise objects affect the algorithm generalizing ability. Keywords: compactness measures, spans of classes, noise objects, nonlinear transformations

# DATA SCIENCE

## Short Communications

### STOCHASTIC AND DETERMINISTIC METHODS FOR CALCULATING GENERALIZED ESTIMATES OF OBJECTS

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Considered are the stochastic and deterministic methods for calculating the generalized estimates of the class of objects in a sample in a two-class pattern recognition problem. A comparative analysis of the two methods is given. It is proposed to use the values of the generalized evaluation of the object as the value of the membership function in fuzzy logic.

### THE METHODOLOGY OF SEARCHING THE REGULARITIES IN OBJECT'S OWN SPACE

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In this paper considered the problem of determining objects own features space. To find objects own features space used nonlinear mapping the object description on numerical axis. The rules of agglomerative hierarchical grouping of different - type (nominal and

quantitative) features allows to reduce the dimensionality of space and to form objects own features space at the same time. The relationship between the objects builds by own features space of each object.

## CLASSIFICATION OF DOCUMENTS IN RELATION TO THEIR CONNECTEDNESS

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The issue of classifying the Uzbek-language documents on the relationship of communication is considered. So it can be shown how the documents are linked to each others. Words can be seen as a set of features.

## CHOOSING THE OWN SPACE OF AN OBJECT BY THE CRITERION OF COMPACTNESS

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The task of choosing the own space of the object of the training sample is considered. For the selection of informative features using the relationship of objects class on the defined system of hyper beads. Relationship is calculated by measure compactness with values in (0,1].

## ELECTRIC VEHICLE ROUTING PROBLEM WITH TEMPORAL AND SPATIAL CONSTRAINTS

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One of the main impact of charging acceleration on the scheduling, assignment and charging process of Electric vehicles (EVs) is an existing amount of EV energy and EV routing problem (EVRP) as well as. In the work, the EVRP will be introduced with temporal and spatial constraints and energy consumption of EVs. Furthermore, charging and discharging times of the EVs will be integrated into the formal model.

## NUMERICAL SOLUTION OF THE CONNECTED DYNAMIC THERMOELASTIC-PLASTIC PROBLEM BASED ON THE DEFORMATION THEORY AND THEORY OF PLASTIC FLOW

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The study of the processes of deformation of structures and their elements, taking into account thermal and mechanical factors, plays an important role in many applied problems

of science and technology associated with the heating of various parts of the object under study. These processes are usually conveniently formulated as connected or uncoupled thermoelastic and thermoplastic boundary value problems. Unrelated problems of thermoelasticity and thermoplasticity and their numerical solutions are well studied in the literature [2, 4, 5, 7].

The related problems of thermoelasticity and thermoplasticity are an important and rapidly developing direction of the mechanics of a deformable solid [1]. In the general case, the coupled boundary value problem consists of the equation of motion of the solid body considering in combination with the equation of heat influx. It should be noted that the system of differential equations in partial derivatives, consisting of three equations of motion and one equation of heat influx, depending on the three components of displacement and temperature, are, respectively, hyperbolic and parabolic. In general, the system of equations is non-linear and complex research. Even in the one-dimensional case, the coupled thermo – elastic problem does not lend itself to an analytical solution, except for some isolated particular cases [8].

The proposed article on the deformation theory method is devoted to the numerical solution of the three-dimensional dynamic coupled problem of thermoelasticity for an isotropic parallelepiped. On the basis of the finite difference method, explicit and implicit grid equations are written [6]. The sweep method [3] is used to solve implicit grid equations. On the basis of two numerical methods, graphs were constructed characterizing the change in displacements and temperatures relative to time and coordinate axes. Comparisons show that the numerical results of the coupled thermoelastic problem, obtained on the basis of explicit and implicit schemes, are quite close.

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